

ISSN 1898-6447

Uniwersytet  
Ekonomiczny  
w Krakowie

# Zeszyty Naukowe

Cracow Review  
of Economics  
and Management

Nr 10 (970)

Kraków 2017



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Ministerstwo Nauki  
i Szkolnictwa Wyższego

Recenzowanie artykułów w języku angielskim – zadanie finansowane w ramach umowy 799/P-DUN/2017 ze środków Ministra Nauki i Szkolnictwa Wyższego przeznaczonych na działalność upowszechniającą naukę

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ISSN 1898-6447

Wersja pierwotna: publikacja drukowana

Teksty artykułów są dostępne na stronie internetowej czasopisma:

[www.zeszyty-naukowe.uek.krakow.pl](http://www.zeszyty-naukowe.uek.krakow.pl) oraz w bazie CEEOL

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Zam. 155/18

# Spis treści

Joseph Yaw Abodakpi <b>Foreign Direct Investment in Ghana – the Political Economy Perspective</b> .....	5
Michał Boda, Radosław Ciukaj <b>Selected Aspects of the Development and Impact of Cryptographic Currencies on the Stability And Functioning of Financial Systems</b> .....	21
Alina Klonowska <b>The Impact of European Funds on Public Expenditure in Poland</b> .....	37
Marta Kornafel, Anna Denkowska <b>Producers’ Optima in Schumpeterian Evolution</b> .....	55
Paweł Oleksy, Marcin Czupryna <b>Market Commentaries and Stock Prices in Poland: A Text Mining Approach</b> .....	67
Kamila Prucia <b>Partial Effectiveness of Scientific Projects Financed through Public Funds</b> .....	79
Agnieszka Rygiel <b>Super-replication of European Options with Convex Payoff under Proportional Transaction Costs</b> .....	91



| *Joseph Yaw Abodakpi*

# Foreign Direct Investment in Ghana – the Political Economy Perspective

## Abstract

Every developmental and economic agenda must seek to attract FDI. Both developed and emerging economies put in place various institutional, legal frameworks and other mechanisms to implement and execute investment agendas. This paper discusses Ghana's situation by examining various programmes the country's government has implemented and institutions that have been established to foster inward investment. Institutions like the Ghana Investment Promotion Centre (GIPC), the Gateway Programme and other reforms within institutions are also presented. The article concludes with a brief overview of the current investment trends in Ghana, the relevance of FDI and the importance of policy direction to protect sensitive sectors of the Ghanaian economy.

**Keywords:** foreign direct investment (FDI), political economy, economic growth, Ghana.  
**JEL Classification:** F21, F55, F59.

## 1. Introduction and Background

The main aim of this article is to lay emphasis on the importance of incentives to attract investment. The existence of an efficient political economy towards foreign direct investment is an undisputed fact. In every developmental and economic agenda, the attraction of foreign direct investment (FDI) seems inevitable to ignore,

as both developed and emerging economies implement various institutional, legal frameworks, and other mechanisms to execute investment agendas for their various economies. This paper discusses the Ghanaian situation by examining various programmes and reforms the government has implemented and institutions that have been established to foster inward investment. The article examines institutions like the Ghana Investment Promotion Centre (GIPC), the Gateway Programme and other institutional reforms. It also discusses the results of some reforms and touches on the importance of joining international bodies like the World Trade Organisation (WTO). The paper ends with a brief overview of the current trend of investment in Ghana and concludes with a look at the relevance of FDI as well as the importance of policy that protects sensitive sectors of the Ghanaian economy.

Countries strive to make their economies competitive and efficient in order to encourage investment from multinational corporations. This approach requires effective institutions and the existence of legal and institutional frameworks to settle disputes among trade partners. The level of transparency and efficiency in host economies is also important as it will help attract foreign and human capital, and also build investor confidence. This will increase not only portfolio investment but also direct investment in the long term. Additionally, a stable and democratic environment will attract voluminous FDI, boosting economic growth. This makes the argument for political economy of foreign direct investment very important and relevant in today's 21st global century. The theme of whether or not political regimes affect the inflow of foreign capital has been the centre of discussion in the political science literature. Authors like G. A. O'Donnell (1978) suggest that authoritarian leaders, in their quest to pursue industrial policies, are more effective than their democratic counterparts at safeguarding multinational corporations' interests. However, no existing evidence shows that authoritarian regimes attract more U.S. foreign investment (Oneal 1994). International investment in the form of foreign direct investment and portfolio investment stimulates the economic growth and development of any economy, a fact upon which a good deal of research has focused, much of it particularly on developing and emerging economies. Economic growth and sustainable development are influenced by both endogenous and exogenous factors, of which foreign direct investment forms a huge part.

Aside from the inflow of foreign capital and the growth it induces, there is also a need for efficient and effective institutional development to complement these efforts. The development of a cautious and efficient financial system is crucial as it acts as a channel for growth. Economic growth can be financed and requires the creation of institutions to implement policies to enhance investment within an economy. Various economic and political reforms are used in both developing and emerging economies, particularly middle income countries like Ghana.

The Ghanaian economy has experienced massive inflow of foreign capital as a result of some of the factors laid out above.

## 2. Theoretical Review

Numerous empirical studies have examined the impact of FDI and financial sector development on economic growth. More and more have been done since the emergence of endogenous growth theory. The empirical studies have thus far yielded mixed results on whether FDI contributes positively to economic growth (e.g. Balasubramanyam, Salisu & Sapsford 1996, Borensztein, De Gregorio & Lee 1998, Carkovic & Levine 2002, Hansen & Rand 2006). In the financial development-growth literature, however, the empirical results have been more conclusive. Most studies have found that financial sector development boosts economic growth (e.g. Christopoulos & Tsionas 2004, King & Levine 1993, Levine, Loayza & Beck 2000). More recent studies have shown that the growth impact of FDI will depend on the extent of financial sector development in host countries (Hermes & Lensink 2003, Alfaro *et al.* 2004). Despite this, the number of empirical studies examining this complementary impact has been relatively small. Most of the cross-sectional empirical studies have not been region-specific, except for the study by M. Omran and A. Bolbol (2003) which focuses on Arabic countries. Few studies have examined the role of financial sector development in enhancing the contributions of FDI on economic growth in Ghana, which has undergone rapid economic growth over the last decade.

Recent developments in growth theory have considered various sources of long-run growth, each of which involves an externality associated with some activity. Examples include human capital accumulation through either learning by doing or education and technological advance through R&D activities. Policy makers and academics contend that FDI can have important positive effects on a host country's development effort, but that empirical evidence for it generating positive spillovers for host countries is ambiguous at both the micro and macro levels. In a recent survey of the literature, G. H. Hanson (2001) argues that evidence that FDI generates positive spillovers for host countries is weak. But V. N. Balasubramanyam, M. Salisu and D. Sapsford (1996) found that when developing countries pursue outward-oriented trade policies, FDI inflows are yield faster growth than in those developing countries that pursued inward-oriented trade policies.

In their paper on the effectiveness of FDI on economic growth, N. Erum, S. Hussain and A. Yousaf (2016) concluded that although FDI is important, it is not more so than domestic investment. They argued that the effectiveness of domestic capital is more reliable and consistent with reference to economic



growth. The loss of government revenue due to fiscal decisions on tax exemptions is noted as a drawback of FDI. Other studies identifying bidirectional causality between FDI and economic growth include S. Naz, G. M. Sabir and A. Mamoon (2015), who concluded that FDI has a positive effect on economic growth while inflation, among other factors, hamper it. I. Ullah, M. Shah and F. U. Khan (2014) found a “bidirectional causal association between domestic investment and FDI whereas domestic investment causes economic growth and economic growth causes FDI”. This recent work throws more light on the importance of domestic investment as a good and relevant source of investment. N. Chayawisan (2015), Z. U. Rahman (2014), A. M. M. Mustafa and S. Santhirasegaram (2013), D. Saqib, M. Masnoon and N. Rafique (2013), J. E. Chen and S. A. M. Zulkifli (2012) all find a positive bi-directional relationship between FDI and economic growth. C. Jude and G. Levieuge (2014) stress the importance of effective institutional bodies, for without them, the effects of FDI in developing economies will not be possible.

### **3. Economic Outlook – the Benefits of Economic and Institutional Reforms**

Belonging to and appending signatures to international institutions, trade-related treaties attract volumes of FDI in various forms (Büthe & Milner 2008). WTO rules, International Monetary Fund (IMF) agreements and numerous other trade agreements ensure that individual member countries stay committed to a well-defined set of liberal economic policies, thus preventing deals from becoming obsolete and ensuring individual member countries’ commitments are dependable. Research J. Goldstein and L. L. Martin (2000) determined that one benefit of international institutions is that they create disincentives for states to behave opportunistically by renegeing on trade agreements and acting unilaterally. International institutions like the WTO and other bilateral trade agreements enshrine provisions and provide remedies for all parties to a trade deal.

Authors including Schiff and Winters (1998) also note that international institutions ensure potential gains from economic growth, increased investment and location advantages. For FDI to be successful, there must be a conscious effort on the part of governments to continue policies and programmes and to reassure investors when regime change occurs. This insurance insures investors against nationalisation and the freezing of assets (Fernandez & Portes 1998, Simmons 2000, Büthe & Milner 2008). With regard to integration agreements, M. Blomström and A. Kokko (1997) point out the serious challenges posed by regional integration and the different dynamics of FDI. M. Busse (2003) concluded that democratic regimes tend to attract FDI. N. M. Jensen (2003), in contrast, believes democratic institutions

tend to inhibit flexibility *vis-a-vis* the implementation of domestic policy. Jensen's voice is not singular: There is a host of other studies suggesting that democracies do not really attract FDI (Yang 2007). Nonetheless, O. Li and A. Resnick (2003) assert that under democratic regimes investors are safe from asset nationalisation and seizures but are entitled to their property rights. Levels of deregulation and home-grown (domestic policies) policies in various economies have encouraged and attracted foreign investors. Openness to trade has become a pivotal point in most economic agendas as globalisation involves voluminous investment. To enable and boost FDI, governments and other investment authorities negotiate international agreements, memorandums of understanding and sometimes – preferential trade agreements to pull in multinational corporations. These agreements normally are signed within the scope of membership in organisations like the WTO and the General Agreements on Tariffs and Trade (GATT), whose guidelines signal a good outlook for investors.

To become part of these international institutions and reap the benefits of international trade, countries must ensure they have the requisite institutions and infrastructure in place both domestically and internationally. In reforming its economy, Ghana introduced and implemented the Structural Adjustment Programme proposed by the IMF in 1983, and two years later joined the WTO. Political and economic reforms – including the banking and financial industry – continue in order to meet internationally accepted standards and attract investors (Antwi-Asare & Addison 2000). These changes have generated unprecedented rates of economic growth and made Ghana a major investment destination. The country has maintained strong trading ties with trading partners including the United States and the countries of the European Union and the African Union. It has also become a strong influential member of sub-regional trading blocs, namely the Economic Community of West African Countries (ECOWAS) and the New Partnership for Africa's Development (NEPAD) (see more: <http://www.nepad.org/about>, accessed: 4.08.2017) and also recently on the debate on signing an Economic Partnership Agreement with the EU (<http://www.myjoyonline.com/opinion/2014/May-15th/ecowas-epa-and-wto-compatibility.php>, accessed: 4.08.2017).

Since 1983, Ghana's financial sector has been restructuring and undergoing transformation. These have included reforms in the financial sector in 1983 under the Structural Adjustment Programme, the Financial Sector Adjustment Programme (FINSAP) in 1988 in two phases, financial liberalisation in 1990 and the introduction of universal banking in 2003. All of these sought to build and create a vibrant financial architectural framework facilitating the transition to a market economy. While FINSAP made gains in the banking sector, it is also credited with establishing Ghana's capital market. Further boosting investor confidence, the Ghana Stock Exchange that was incorporated in July 1989 as a private entity

under the Companies Code of 1963 (Act 179), went public in April 1994 to ensure proper governmental and institutional oversight of its trading activities.

Pushing for full liberalisation in the financial sector, “universal banking” was introduced by the Bank of Ghana in the first quarter of 2003. This allowed banks to embark on commercial, development, investment or merchant banking without separate requirements to operate in these new areas. Financial sector development coincided with a period of strong growth. Gross domestic product increased from  $-7.5\%$  in 1982 to  $6.2\%$  in 2006, while per capita income increasing from  $-10.2\%$  to  $4.3\%$  over the same period (World Development Indicators 2008). These reforms have boosted financial sector effectiveness and efficiency, resulting in positive economic growth since the start of the economic adjustment period and consistently since 1990, especially from 2001 to 2013.

Ghana’s economic growth rate and expansion is not only a regional concern but has implications on international decision-making processes regarding investments in Africa. Some of the major drivers of the Ghanaian economy include mining, agriculture, the petroleum (oil and gas discovery), and financial services, a sector that has grown as the number of banks has increased. For investments to yield the desired returns, energy production must play a central role in the economy. In Ghana, the completion of the Bui hydroelectric dam in 2013 together with gas production in Atuabo, which started in 2014, has enhanced the supply of electricity for production purposes. This in turn has improved manufacturing and services sector output. The robust nature of the financial sector and continuous improvement in the telecommunications and transport sector remain positive drivers, and a strong basis for attracting investment. Another advantage Ghana possesses is its political stability, good governance, and respect for the rule of law and good business environment.

The economic recession of 2008 caused economic growth to slow, but growth continued after reforms and other government efforts at fiscal consolidation were made. Monetary and fiscal policies implemented in the wake of the 2008 financial meltdown and other implemented policies had a positive impact on infrastructure and also increased investment. Ghana’s strong export regime also boosted the economy, with major trading partners in Southeast Asia, the US and the EU importing more. The economy later made strong progress thanks to fiscal consolidation lowering the fiscal deficit of  $6.3\%$  in 2015. While the economy missed its fiscal target of  $5.3\%$  of gross domestic product in 2016, it achieved  $3.6\%$  more than the  $3.3\%$  projected. The inflation rate of  $15.4\%$  in December 2016 fell to  $13.3\%$  in January 2017, closer to the  $6\text{--}10\%$  range the Central Bank was calling for (<http://www.worldbank.org/en/country/ghana/overview/>, accessed: 1.07.2017). These figures demonstrate Ghana’s prudent economic management.

Even with global weakness and imbalances, together with weak global commodity prices, Ghana's external balance improved in 2016, reflecting increased exports and falling imports. The current account deficit narrowed to 6.4% of GDP in 2016 from 7.6% of GDP in 2015. Foreign reserves surged from 4.4 billion USD to an estimated 4.9 billion USD, boosting Ghana's balance of payment accounts (<http://www.worldbank.org/en/country/ghana/overview/>, accessed: 1.07.2017). Furthermore, bolstered by improved oil and gas production, increased private-sector investment, improved public infrastructure development and sustained political stability, growth is still expected to be positive (<http://www.afdb.org/countries/west-africa/ghana/ghana-economic-outlook/>, accessed: 5.08.2017). These macroeconomic developments are all expected to encourage investment in the long term.

Ernest and Young (E&Y) recently cited the Ghanaian economy as new a regional hotspot for global investment and doing business (<http://www.ey.com/GL/en/Newsroom/News-releases/News-foreign-direct-investment-in-sub-saharan-africa-on-the-rise>, accessed: 4.08.2017). Massive investment holdings have been held by major investors, i.e. The United States, the countries of the EU, the United Kingdom and China. Investment is expected to increase thanks to the favourable conditions and economic stability that continues to flourish in the country. The emergent Ghanaian economy has stepped into the global spotlight alongside other major regional competitors Nigeria, South Africa and Kenya, all of which compete for investments on the African continent. Considerable FDI over the decades has strengthened Ghana's economy and increased its financial markets' base of capital. Other financial and banking sector reforms in addition to equity markets developments keeps investors attracted to the Ghanaian economy.

With the country attaining a lower middle-income status (<http://www.imf.org/external/pubs/ft/survey/so/2013/car061213a.htm>, accessed: 5.08.2017), managing the economy quite effectively and maintaining a stable political environment, Ghana has had a positive impact on Africa's economic outlook. These achievements translate into advantages for some African economies, while also having significant implications for western policy towards the continent's developmental agenda. In a critical starting point, it is relevant to understand the diverse impact of the inflow of FDI and other forms of economic assistance to the African continent. The majority of African economies are being affected indirectly by the phenomenal growth of Asian economies (particularly China's) and their increased trading quota with the continent.

It is clear that African countries must not give up on their growth momentum and their determination to tackle developmental problems, nor on attaining the renewed vision of a prosperous, vibrant region as captured partly in the Millennium development goals, which include halving extreme poverty rates

especially (see more: <http://www.un.org/millenniumgoals/>, accessed: 5.08.2017). From a policy perspective, the establishment of the China-Africa Forum offers unconditional support for the African Union and other various programmes, including the New Partnership for African Development (NEPAD), which is being integrated into the sub-regional and national development strategies to attract more investment. Furthermore, economic and trade agreements with the European Union and United States must be negotiated in good faith to have value for the continent's vast natural resources, which are the main driving force for growth and investment on the continent.

## **4. The Political Economy of Foreign Direct Investment in Ghana**

### **4.1. FDI and Trade Policies**

Government policies and rules regarding the investment climate in a given economy is of utmost importance to investors. Both advanced and emerging economies regard investment, including FDI, crucial to the development processes. FDI significantly stimulates economic growth in both developed and emerging economies. The capacity of an economy to absorb the vast benefits and spillover of FDI depends on the capital base and the ready availability of skilled labour to apply modern technology in production processes and the delivery of services.

### **4.2. Ghana's Legal Framework and Investment Policy**

The Promotion Act (Act 478) of 1994 established the Ghana Investment Promotion Centre (GIPC), with the aim of attracting foreign and domestic business and monitoring their business operations. The GIPC encourages, promotes and facilitates investment in Ghana consistently with the country's economic policy goals and also registers projects. China's investments benefit immensely from Ghana's good infrastructure and financial sector development. The investment climate and better legal and administrative nature of the economy makes doing business straightforward. Between 1982 and 1992, Ghana implemented the Structural Adjustment Programme (SAP)<sup>1</sup>, which was accompanied by various transformations, including the signing of various trade treaties and agreements with the WTO, and also the liberalisation of the financial sector, which encourages foreign investment. Ghana's liberal FDI policy makes the country

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<sup>1</sup> See more: <http://dspace.knust.edu.gh:8080/xmlui/handle/123456789/2969> (accessed: 5.08.2017) – for detailed information on the SAP implemented from 1982–1992, see the Ministry of Finance and Economic Affairs.

a safe destination for investment opportunities, and moves the wealth of its natural resources to the fore.

Investment regulations allow foreign investors to employ foreign nationals in Ghana. The GIPC's investor-targeting strategy has identified China as one of ten countries with the potential to increase investment in Ghana and plans to open an office in Shenzhen, a sub-provincial city in Guangdong province, to facilitate investment between China and Ghana. Investment forums and missions staged in both Ghana and China will now implement development plans to improve language integration, drive investment and promotional activities (see more: *Looking East...* 2009). One also has to state that Ghana enjoys high levels and volumes of investment from other developmental partners including the UK, the US and the EU.

### **4.3. Special Economic Zones and the Gateway Programme**

In 1995, the Parliament of Ghana promulgated the Free Zones Act that was later signed in to law. This Act of parliament was intended to accelerate the exploration of the country's potential. In order to assist the activities of Ghana's export processing zones (EPZs), the Ghana Free Zones board was established to help monitor activities. The EPZs are aimed in particular at the industrial sector, located physically and administratively without and outside the jurisdiction of the customs barrier. Ghana's special economic zone is focused largely on export production. This zone helps attract investors into various sectors by providing incentives through an administrative and policymaking body, when it relates to investment. The main objective of the Investment Act is to attract FDI. To achieve these investment objectives, there are some provisions allowing for: a ten-year tax exemption on income or profit; a minimum tax of 8% after 10 years of business operation; tax withheld on dividends accruing from free zone investments; and the opportunity to hold 100% of the shares in any free zone venture. Finally, the Investment Act also assures the repatriation of earned profits and prevents the nationalisation of assets in the event of government or regime change. All these elements and policies are intended to facilitate the conducting of business, boost investor confidence and foster a positive investment climate.

Another crucial policy tool to enhance and attract investment is the NEPAD (Abodakpi 2012, p. 34–35). Intended to position Ghana as an investment and trade destination hub in West Africa, this gateway programme emphasises export processing zones and FDI as it seeks to attract financial services, transport and telecommunication networks that will facilitate the transfer of processed and finished goods for exports. The emphasis is on developing three export-free zones by private operators, while the government provides the framework for investment to advance the programme. The Programme is also focused on

ISO 9000 and International Chamber of Commerce (ICC) standards (Abodakpi 2012, p. 34–35).

## **5. Investment Motives**

For multinationals, the most important and common incentives to motivate an investment are mainly: seeking efficiency, seeking resources and seeking markets (Dunning 1993, p. 78). Adequate availability of natural resources combined with unskilled, skilled and semi-skilled labour and the presence of a well-developed infrastructure encourages resource-seeking investment, as in the case of China in most of Africa and other parts of the world. The abundance of raw materials provides a comparative advantage and creates the possibility of trade. Resource seeking investment takes place rapidly when there is a lack of an adequate capital base to develop the vast base of resources or when there is a shortage of skilled and professional knowledge to apply (UNCTAD 1998).

The process of investment is encouraged when there exists a huge addressable market in terms of population size and per capita income. New unsaturated markets give firms the chance to compete and grow, thereby making specialisation a goal in gaining economies of scale. In assessing the size of markets, China sees the Sub-Saharan African corridor as one massive investment hub and other regional blocks as investment destinations. Individual countries including Nigeria, Ghana, Kenya, South Africa and other investment hotspots provide markets for China's manufacturing and production capabilities.

Finally, the motivation to efficiently seek FDI is "to rationalise the structure of the established resource base or market-seeking investment in such a way that the investing company can gain from the common governance of geographically dispersed activities. The point in efficiently seeking MNE is to take advantage of different endowments, cultures, institutional arrangements, economic systems and policies and market structures by concentrating production in a limited number of locations to supply multiple markets. To seek foreign production efficiently, cross-border markets must be both well-developed and open, so regionally integrated markets tend to flourish" (Kudina & Jakubiak 2008). It should also be noted that multinationals do combine all the above motives when seeking investment proposals.

## **6. Current Trends of Foreign Direct Investment in Ghana**

The inflow of FDI volumes has been steady in the period following the global recession of 2008, but overall "(...) Ghana has been one of the world's fastest-growing economies, on the back of the emergence of its oil industry and underpinned

by its political stability and the strength of its other commodity sectors. While GDP growth is expected to slow this year, the Ghana Investment Promotion Centre (GIPC) is targeting a 20% increase in foreign direct investment (FDI), following a dip in 2013” (<http://www.oxfordbusinessgroup.com/news/ghana-targets-20-fdi-increase-2014>, accessed: 6.08.2017). This trend will likely continue to be positive. The World Bank’s “Doing Business 2014” ranks Ghana 67th out of 189 countries – one of the best performing economies in Africa and above some European Member States. Ghana’s business environment has improved to 28th in the world for accessing credit facilities and 34th for investor protection (<http://www.oxfordbusinessgroup.com/news/ghana-targets-20-fdi-increase-2014>, accessed: 6.08.2017).

Table 1. Top 10 Source of Investment by FDI Value (2017)

Countries	FDI Value (Million USD)	FDI Value (%)
France	127.83	64.75
Others (not specified)	20.00	10.13
China	17.80	9.02
USA	9.06	4.59
Mauritius	5.98	3.03
Switzerland	5.50	2.79
India	3.22	1.63
Netherlands	3.20	1.62
Turkey	2.72	1.38
South Africa	2.10	1.06
Total	197.41	100.00

Source: the author’s own calculations based on (GIPC 2017a, p. 3).

FDI accounts for 203.74 million USD (98.55%) of total value, while about 3 million USD (1.45%) accounts for the local currency component with total foreign equity amounting to 36.32 million USD. During the second quarter of 2017, 19.64 million USD was also realised as an initial equity transfer. Analysing the countries with numerous projects, China still tops the list with 11 projects, while France is first by estimated value of projects, at 127.83 million USD (for further details see GIPC 2017b).

In the second half of 2017, the total number of projects registered was 46, 35 of which were foreign owned, representing 76% and with an estimated value of 61.75 million USD accounting for almost 30% of the total value of projects. Of these investments, 11 were joint ventures between Ghanaian and foreign partners with an estimated value of 144.98 USD. The distribution of registered Ghanaian projects also saw significant improvements, with 20 investments across



Table 2. Top 10 Investor Countries by Value of FDI (2017)

Countries	FDI Value (Million USD)	FDI Value (%)
Netherlands	2,437.60	77.85
India	405.36	12.95
France	128.83	4.11
United Kingdom	49.71	1.59
China	44.84	1.43
Others (not specified)	20.00	0.64
USA	15.39	0.49
Canada	12.08	0.39
Kenya	9.19	0.29
Mauritius	7.98	0.25
Total	3,130.98	100.00

Source: the author's own calculations based on (GIPC 2017a, p. 6).

ten regions of the country. The capital city of Accra registered the highest number of projects, at 14, with the service sector registering eight of those. The economic benefits of these projects also have created about 497 jobs that will also affect the national GDP. China has managed a place in the top 10-investor countries since establishing trade and economic relations with Ghana (GIPC 2017b).

## 7. Conclusion and Summary

Within the legal and institutional framework for investment in Ghana, the GIPC promotes investment and is mandated to promote and attract foreign direct investment to the country. The contribution of FDI to Ghana's yearly GDP cannot be underestimated, hence there is a need to put institutions and mechanisms in place to facilitate the attraction and retention of all forms of investment, from medium to long-term ones.

The effects and implications of attracting FDI bears a diversity of outcomes. Employment of both skilled and unskilled labour has been a major positive factor in Ghana. Training and technological transfers have been made available, together with spillovers into the various sectors. The employment of skilled experts boosts the human development index in the long run, as efficient methods are introduced and scarce resources of production are utilised properly. On the other hand, it is clear that the quantity of commodities exported from Ghana to other trading partners is abysmally lower than the percentage and volume of imports to Ghana from China (see more: *Looking East...* 2009). The manufacturing sector of the

Ghanaian economy and other African countries like South Africa and Nigeria are missing out on trade opportunities with China and other Asian economies.

Because of the increased and growing investment from Asia, especially Chinese business and investment with majority shareholder control, the industrial sector is challenged by the import of highly subsidised manufactured goods. This competition poses both internal and international challenges to Ghana's total output and the pricing of its goods and services. The relative smaller size of the Ghanaian economy and the stiff competition it faces due to globalisation, market liberalisation and deregulation policies should not be ignored by policymakers. Governments must put policies and regulations in place to protect infant and growing industries in all sectors of the economy.

Another area of serious concern is Ghana's textile industry. The sector faces unequal competition, due to heavily subsidised and inexpensive Chinese textiles flooding the markets, making it difficult to sell quality products produced in Ghana. Policy directions towards certain sensitive areas of the economy should be encouraged to protect and enhance both the development of local industries and national pride. A closer examination of investment trends in most African and other emerging economies clearly shows why China is interested in trade, and it is for numerous reasons. Economic interest in the continent's vast natural resources, including raw materials for the textile industry, metals and the recent discovery of petroleum fields increases the appetite not only for Chinese investment but also for also western multinationals. Intra- and inter-African trade and investments should be encouraged, as this will boost confidence in the continent's efforts to develop.

One very recent important challenge for the Ghanaian economy has been the proper management of the country's currency against other major currencies. The recent devaluation of the Cedi due to huge trade and current account imbalances poses problems for both domestic and international investors. For instance, in 2013 the currency plummeted 15% against the USD, making it one of the worst performing currencies at that time. Fortunately, it has since stabilised. Numerous measures and mechanisms have been implemented to protect the currency and boost investor confidence in the economy after a series of weak exchange rate positions in recent years. Despite all these setbacks, the country remains one of the largest FDI recipients in West Africa.

Finally, is a fact that FDI induces economic growth and expansion through various channels, enhances capital formation and transfer technology and increases competition. Economies must also be cautioned against relying too much on FDI-led growth. Governments and institutions must seek alternative sources of growth to diversify their economies and help prevent against external negative shocks. Economies must develop the legal structure that will create and foster a healthy business environment. When local and national business thrives, it is

easy for multinationals to enter well performing economies, and they are induced to do so. Governments must therefore provide better incentives for local producers to bring out quality products and drive prices down for better competition on the international stage. The development of inward FDI should be the main agenda for most economies, especially in emerging and developing economies. To benefit fully from all of the policies implemented to attract FDI, economies must have the absorptive capacity and human capability to effectively and efficiently accumulate all of the positive spillover effects.

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## **Bezpośrednie inwestycje zagraniczne w Ghanie z perspektywy ekonomii politycznej** (Streszczenie)

Każdy kraj, bez względu na poziom rozwoju (rynki rozwinięte, wschodzące czy rozwijające się), wprowadza w swojej polityce różnorodne ramy prawne i instytucjonalne oraz inne formy rozwiązań, w celu wdrożenia i realizacji własnych planów rozwojowych i inwestycyjnych. Zauważyć można, że stabilna gospodarka i sukcesywne wprowadzanie przyjaznych rozwiązań przyciąga inwestorów oraz przyczynia się do wzrostu bezpośrednich inwestycji zagranicznych. Celem artykułu jest omówienie przypadku Ghany, poprzez analizę różnych programów wprowadzanych przez rząd oraz przegląd instytucji powołanych w celu wspierania wewnętrznych inwestycji. Przedstawione zostały m.in. takie instytucje, jak Ghana Investment Promotion Centre (GIPC), centrum promocji inwestycji w Ghanie, the Gateway Programme (tj. program centrum inwestycyjnego), a także przeprowadzane reformy strukturalne. Omówione zostały również obecne trendy inwestycyjne w Ghanie, znaczenie bezpośrednich inwestycji zagranicznych oraz kierunki polityki gospodarczej mającej na celu ochronę kluczowych sektorów ghańskiej gospodarki.

**Słowa kluczowe:** bezpośrednie inwestycje zagraniczne (BIZ), ekonomia polityczna, wzrost gospodarczy, Ghana.

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# Selected Aspects of the Development and Impact of Cryptographic Currencies on the Stability and Functioning of Financial Systems

## Abstract

The first cryptographic currency (Bitcoin) was created in 2009. Since then, cryptocurrencies have undergone dynamic development, and their user numbers risen apace. This article analyses and estimates the impact, development and growth of cryptographic currencies on the stability and functioning of financial systems. The paper also examines whether cryptocurrencies fulfill the basic functions of money. The first part addresses technical and legal issues of cryptographic currencies. The second performs a statistical analysis and impact assessment on public finances, banking sector as well as individual users have been performed. Bitcoin quotes are compared to other traditional financial instruments including rate of return, coefficient of variation, range, and Pearson's correlation coefficient. The analyses which have been conducted lead to the conclusion that cryptocurrencies cannot be treated equally with money (they are treated more as investment assets), they are far more volatile than traditional financial instruments, and there is a significant similarity between cryptographic currencies and pyramid schemes.

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**Keywords:** cryptographic currencies, Bitcoin, stability, financial system.

**JEL Classification:** G14, G20, O33.

## 1. Introduction

With the advancement of the computerisation, as can be seen in recent decades, more and more aspects of everyday life are being transferred to the realm of virtual reality. As a result, numerous unknown phenomena have emerged, including economic ones. Cryptocurrencies are among the latest products of human ingenuity of recent years, and function solely in the virtual realm. They are intended to be instruments that are not subject to any administrative regulation, are independent of central banks, and are valued only by the market (Dopierała & Borodo 2014, s. 1).

Given that one of the main functions of cryptocurrencies is to enable payments, national and international administrators must determine whether cryptocurrencies can be treated as money at all. The answer to this question is not an easy one, given the functions of money in the economy. These include (Podstawka 2013, p. 44):

a) as a medium of exchange – money effectively eliminates the double coincidence of wants problem by serving as a medium of exchange that is accepted in all transactions, by all parties, regardless of whether they desire each others' goods and services,

b) as a unit of account – money is a common standard for measuring the relative worth of goods and services,

c) as a store of value – money's value can be retained over the time, so it is a convenient way to store wealth.

In November 2008, a prospectus (sometimes called a manifest) was posted by a person or group of people working under the name of Satoshi Nakamoto, who expressed the need to create a new currency, based entirely on a peer-to-peer system (P2P). P2P is a communication model in a computer network that provides all participating devices with the same data sharing as the client – server architecture. This currency would allow online payments to be made directly between users, bypassing institutions or financial intermediaries. What is more, the payment system, connected by virtual currency, would be based on cryptography rather than on trust in a third parties ([www.bitcoin.org/bitcoin.pdf](http://www.bitcoin.org/bitcoin.pdf), accessed: 14.08.2017). This is how Bitcoin (BTC), now considered the first and most common cryptocurrency, was created. Nakamoto also raised a heated discussion about contemporary forms of money.

## 2. Research Methodology

Today, there exists a large-scale discussion about the legitimacy of the cryptocurrencies (particular Bitcoin) as a means of payment or as an investment instrument. The payment function best reflects the idea that formed the foundation for the cryptocurrencies. However, literature studies and analysis of statistical data clearly indicate that the investment function is the main premise for acquiring cryptocurrencies. The purpose of this article is to present the relationship between cryptocurrencies and other currencies and to show the impact of cryptocurrencies on public finances, the banking sector and individual users. It will also discuss the dark side of cryptocurrencies and their similarity to financial pyramid schemes. It outlines the prospects for development in the context of their payment functions. The article starts with the theoretical aspects of cryptocurrencies, including the technical and legal conditions of their functioning. Rate of return, coefficient of variation, range and Pearson correlation coefficient have been used to analyse Bitcoin quotes in relation to selected national currencies, commodities and stock indexes.

## 3. Cryptographic Currencies and Forms of Modern Money

The name “money” comes from Latin (*pecunia*) and means horn cattle, which in Roman times was used as a means of exchange. This suggests that anything can be established as money, relying solely on the principles of social contract. Regardless of its external form and economic system, money is defined today as a legally defined, commonly accepted means of payment that can express, store and accept values, and whose value is closely linked to real Gross Domestic Product (GDP) (Schaal 1996, p. 26).

Money can be classified according to different criteria. While the forms of money have included primitive, metallic, banking and electronic (Piaszczyński 2004, p. 19), contemporary money exists in two main forms:

- a) cash – banknotes and coins,
- b) deposit money – the subject of accounting records of banks and has no physical form.

New types of payment instruments have emerged which could easily replace the current forms of money. From this perspective, cryptocurrencies can be seen as an alternative to both cash and deposit money. However, any good that would fulfill the role of money must meet the following criteria, e.g. (Bala, Kopyściński & Srokosz 2016, p. 57):



- a) durability – it needs to last,
- b) portable – is easy to carry, convenient and easy to use,
- c) divisible – it can be broken down into smaller denominations,
- d) hard to counterfeit – it can not easily be faked or copied,
- e) must be generally accepted by a population,
- f) valuable – generally holds value over time.

By analysing the technological aspects of the creation and operation of cryptocurrencies, it can be concluded that they fulfill all of the above-mentioned characteristics satisfactorily, and some features are even fulfilled to a greater extent than money in cash or deposit money form. Thus, cryptocurrencies could be accepted as modern forms of cash.

#### 4. Technical Conditions for the Functioning of Cryptographic Currencies – Bitcoin

The Bitcoin payment network is built on the basis of a cryptographic P2P protocol that gives all users equal rights. In other words, users around the world – or, more precisely, their computers – create and control the network they belong to. This means there is no central server responsible for Bitcoin. From the point of view of a regular user, Bitcoin is just a programme installed on a computer or mobile phone, one that allows access to a virtual wallet that sends and receives bitcoins ([www.bitcoin.org/bitcoin.pdf](http://www.bitcoin.org/bitcoin.pdf), accessed: 15.08.2017).

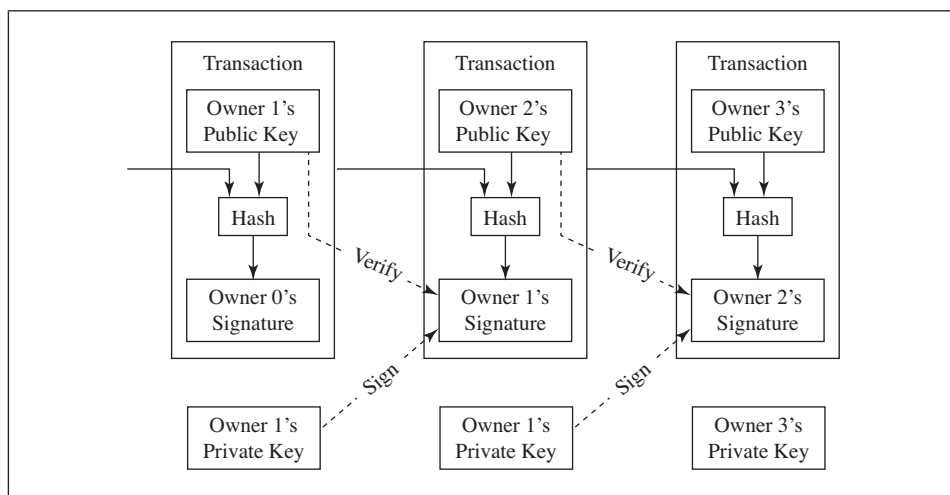


Fig. 1. The Bitcoin Network Transaction Diagramme

Source: (Nakamoto 2017).

The BTC wallet consists of three numbers:

- a) a private key – used to “sign” (authorise) a transaction,
- b) a public key – generated from a private key and serves as an address,
- c) an address – consists of 27 to 34 alphanumeric characters and can be compared to an email address. If a wallet user wants to pay or transfer bitcoins, he or she must provide the wallet address where the corresponding balance is stored.

The Bitcoin network is based on a widely available, scattered database. This is a chronological list of the Bitcoin network and is based on a widely available, scattered database. This is a chronological list of all transactions, otherwise called block chains. Figure 1 illustrates the Bitcoin network transaction diagramme.

A transaction is a message of some value between the addresses. It is executed and authorised using a private key by each user. All current transactions, before approval, are collected and saved in the block every 10 minutes. In order to prevent the double release of bitcoins or forgery, coherence and chronological chains of blocks are based on “Proof of Work” (PoW), or evidence of action taken. This involves calculating, with a certain probability of the hash (a string of letters and numbers used as a shortcut) containing information about current transactions and the hash of the previous block. This task is performed by “miners”. In short, every transaction made between users is confirmed by the P2P network in an expanding process. This process is a kind of competition, because new bitcoins are randomly allocated to users who provide computing power. The greater the computational power a user gives, the greater the probability he or she will receive new bitcoins. Coin allocation follows the condition of “block break”, which means solving a cryptographic puzzle (Dopierala & Borodo 2014, p. 3).

## 5. Legal Aspects of Cryptocurrencies – the Example of Bitcoin

Along with increasing interest in cryptocurrencies among societies throughout the world, legal aspects of the virtual currency have become one of the most important issues. The attitude of authorities to cryptographic currencies in different countries remains unclear. Officially, cryptocurrencies, including Bitcoin, do not – and likely will not for the foreseeable future – have legal or financial definitions. Despite this, some authorities have issued statements defining their status in the light of existing legislation.

In Poland, the Ministry of Finance has used the following legal acts to define Bitcoin:

- a) The Act of 27 July 2002 Foreign Exchange Law (*Ustawa Prawo dewizowe*), Journal of Laws of 2002 no 141 item 1178,

b) The Act of 12 September 2002 on Electronic Payment Instruments (*Ustawa o elektronicznych instrumentach płatniczych*), Journal of Laws of 2002 no 169 item 1385,

c) The Act of 19 August 2011 on Payment Services (*Ustawa o usługach płatniczych*), Journal of Laws of 2011 no 199 item 1175, as amended,

d) The Act of 29 July 2005 on Trading in Financial Instruments (*Ustawa o obrocie instrumentami finansowymi*), Journal of Laws of 2010 no 211 item 1384, as amended.

The Ministry of Finance has declared that operating and trading with cryptocurrencies is not illegal. However, given the lack of universal acceptability, they cannot be considered domestic or foreign currency, nor as a means of payment. Cryptocurrencies may also not be covered by the definition of electronic money or financial instruments. Further, cryptocurrency transactions are not payment transactions, because they only cover payment or cash transfer (Kurek 2015, p. 153).

The European Central Bank issued a detailed report on digital currency in 2012 (<https://www.ecb.europa.eu/pub/pdf/other/virtualcurrencyschemes201210en.pdf>, accessed: 17.08.2017), showing the differences between typical electronic currency and virtual currency. According to the report, the European Union does not classify any virtual currency as electronic money because, according to Directive 2000/46/EC, money is necessarily related to the claim to the issuer to issue an appropriate amount of money, which in the case of cryptocurrencies cannot be stated, because there is no issuer. In 2013, the European Banking Authority (EBA) issued a warning about the possible risks of theft and fraud virtual currencies posed.

Some countries have a favourable attitude towards cryptographic currencies, although with some degree of distrust. The Belgian National Bank, along with other financial institutions, withdrew an official warning statement about investments in Bitcoin. In Germany, Bitcoin is not considered a currency, but is defined as “private money” and included in units of account (<https://bitcoinmagazine.com/articles/regulation-bitcoins-germany-first-comprehensive-statement-bitcoins-german-federal-financial-supervisory-authority-bafin-1391637959/>, accessed: 17.08.2017). In the United States, each state has its own cryptocurrency regulation, though state bodies have acknowledged that Bitcoin’s anonymity may lead to money laundering, terrorist financing or drug trafficking (<http://www.ibtimes.com/bitcoin-new-york-state-continues-its-path-regulating-virtual-currency-1721103>, accessed: 17.08.2017).

## **6. Examples of Other Cryptographic Currencies**

Bitcoin was created in 2008 and by 2011 was the only cryptographic currency in the world. However, in the course of time, Bitcoin derivatives also started to emerge, and often are called altcoins (alternative coins). Most of these coins either have already been forgotten or have very small capitalisation, because they did not bring in any technological innovation. However, at least several cryptocurrency derivatives deserve attention, e.g.:

a) Namecoin (NMC) – the first alternative cryptocurrency, it was launched in April 2011. Mostly based on Bitcoin source code. The difference between these two cryptocurrencies is that Namecoin acts simultaneously as a decentralised DNS (Domain Name System). This is because NMC allows one to register in the chain of blocks one's own Internet domain with the .bit22 ending, so that it is not subject to ICANN supervision (The Internet Corporation for Assigned Names and Numbers supervision). In practice, this allows for domain registrations that cannot be blocked by any authority ([www.wiki.namecoin.info](http://www.wiki.namecoin.info), accessed: 18.08.2017);

b) Litecoin (LTC) – the work of Google employee Charles Lee. It was created in October 2011 and now has the second largest capitalisation (after BTC). The main difference is the change in the Proof of Work algorithm from SHA-256 to Scrypt. This difference raises the security of the network, and the special equipment designed for the expansion of Bitcoin cannot be used to expand Litecoin ([www.coinmarketcap.com/](http://www.coinmarketcap.com/), accessed: 18.08.2017);

c) Primecoin (XPM) – a very interesting currency because its supply is not precisely defined by the values in the source code. The final amount of coins is not well known, because it is based on special numbers of primitive numbers strings, which are simultaneously discovered during expansion ([www.primecoin.io/bin/primecoin-paper.pdf](http://www.primecoin.io/bin/primecoin-paper.pdf), accessed: 18.08.2017);

d) Darkcoin (DRK) – created in 2014, DRK is a relatively new cryptocurrency. Nowadays, in the world of cryptocurrencies, new cryptocurrencies are created with a view to having the most secure, anonymous system. Darkcoin has a special algorithm that is supposed to combine multiple transactions into an anonymous one in order to prevent tracing individual transaction histories.

## **7. Analysis of Bitcoin Volatility Quotes in Relation to Selected National Currencies, Commodities and Stock Indexes**

The high volatility of cryptocurrencies relative to national currencies raises the question of a possible impact on the stability and functioning of financial markets. As the last section made clear, there are many cryptocurrencies on the

market. But for this study Bitcoin (BTC) was selected as it is the most used and recognised one in the world, has the largest capitalisation and trading volumes. BTC quotes in relation to national currencies (EUR, USD, PLN, CNY, GBP) and gold have been compared with quotes of other traditional currency pairs (such as EUR/USD, EUR/PLN, etc.), quotes of selected commodities (gold, Brent oil) and stock indexes (SP 500, FTSE 250, DAX).

The following statistical methods were used: rate of return, coefficient of variation, range and Pearson correlation coefficient. Based on the data in Table 1, BTC was significantly more volatile than other currencies. For BTC, the daily rates of return were in the range of  $-36.25\%$  to  $52.89\%$ , while traditional financial instruments ranged from  $-8.57\%$  to  $9.10\%$ . The situation with the coefficient of variation was similar. The average value of this indicator for BTC was  $28.27\%$ ,

Table 1. Intraday Statistical Analysis of BTC Quotes Relative to National Currencies and Selected Commodities and Stock Market Indices in the Period 19.07.2010–15.08.2017 (in %)

Relation	Rate of Return		Coefficient of Variation <sup>a</sup>	The Highest Value Shift <sup>b</sup>	The Lowest Value Shift <sup>c</sup>	Range <sup>d</sup>
	the Highest	the Lowest				
BTC/EUR	39.97	-22.09	25.41	44.64	-38.33	50.15
BTC/USD	52.89	-36.25	48.14	97.96	-56.57	100.02
BTC/PLN	39.57	-22.48	25.17	44.22	-38.29	49.97
BTC/CNY	21.50	-24.10	16.33	21.70	-29.22	39.49
BTC/GBP	40.19	-22.38	25.55	44.86	-38.73	50.07
XAU/BTC	23.07	-28.62	29.00	60.49	-31.00	66.44
EUR/USD	3.01	-2.90	2.23	3.86	-4.50	4.50
EUR/PLN	2.70	-3.03	2.23	4.39	-3.21	4.73
EUR/CNY	3.06	-1.54	1.95	3.60	-3.14	3.64
EUR/GBP	6.78	-2.05	4.75	9.22	-2.28	9.32
XAU/EUR	9.02	-8.57	7.03	14.19	-9.60	14.19
CB.F USD	9.10	-8.30	5.75	10.31	-9.51	12.11
SP 500	4.67	-6.59	3.81	5.80	-6.61	6.61
FTSE 250	3.58	-7.19	5.98	4.09	-12.25	12.25
DAX	6.05	-7.07	4.65	6.78	-8.14	8.94

<sup>a</sup> calculated as the quotient of standard deviation and the opening price, <sup>b</sup> calculated as the quotient of the highest price and the opening price, <sup>c</sup> calculated as the quotient of the lowest price and the opening price, <sup>d</sup> calculated as the quotient of the difference between the highest and lowest prices and the opening price.

Source: the authors' own elaboration based on: <https://stooq.pl> information (accessed: 15.08.2017) and (Bala, Kopyściański & Srokosz 2016, p. 90).

Table 2. Correlation of Intraday Rates of Return in the Period 19.07.2010–15.08.2017

Relation	BTC/ EUR	BTC/ USD	BTC/ PLN	BTC/ CNY	BTC/ GBP	XAU/ BTC	XAU/ USD	XAU/ PLN	XAU/ CNY	XAU/ GBP	XAU/ EUR	CB.F USD	SP 500	FTSE 250	DAX
BTC/ EUR	1.00	×	×	×	×	×	×	×	×	×	×	×	×	×	×
BTC/ USD	0.58	1.00	×	×	×	×	×	×	×	×	×	×	×	×	×
BTC/ PLN	0.97	0.56	1.00	×	×	×	×	×	×	×	×	×	×	×	×
BTC/ CNY	0.65	0.37	0.62	1.00	×	×	×	×	×	×	×	×	×	×	×
BTC/ GBP	0.99	0.57	0.97	0.65	1.00	×	×	×	×	×	×	×	×	×	×
XAU/ BTC	-0.94	-0.54	-0.91	-0.63	-0.93	1.00	×	×	×	×	×	×	×	×	×
EUR/ USD	-0.08	0.05	-0.08	-0.02	-0.04	0.05	1.00	×	×	×	×	×	×	×	×
EUR/ PLN	0.02	-0.01	0.08	0.03	0.04	-0.02	-0.18	1.00	×	×	×	×	×	×	×
EUR/ CNY	-0.06	-0.01	-0.06	0.02	-0.03	0.04	0.34	0.01	1.00	×	×	×	×	×	×
EUR/ GBP	-0.01	0.04	0.01	0.02	0.09	0.01	0.51	0.06	0.19	1.00	×	×	×	×	×
XAU/ EUR	0.02	-0.01	0.02	0.02	0.01	0.13	-0.30	0.06	-0.08	-0.17	1.00	×	×	×	×
CB.F/ USD	-0.02	0.01	-0.04	-0.03	-0.04	0.04	0.17	-0.20	0.01	-0.03	0.05	1.00	×	×	×
SP 500	-0.01	0.02	-0.03	-0.03	-0.03	0.01	0.25	-0.38	-0.02	0.00	-0.13	0.37	1.00	×	×
FTSE 250	-0.03	0.01	-0.05	-0.05	-0.05	0.02	0.14	-0.36	-0.06	-0.05	-0.10	0.28	0.56	1.00	×
DAX	0.02	0.03	0.01	0.01	0.00	-0.01	0.07	-0.35	-0.08	-0.04	-0.06	0.22	0.56	0.58	1.00

Remark: The blue colour stands for results of correlation between quotes of different BTC pairs, orange for the correlation between BTC and traditional financial instrument quotes, and green for the correlation between various traditional financial instruments pairs.

Source: the authors' own elaboration based on stooq.pl information (accessed: 15.08.2017) and (Bala, Kopyściański & Srokosz 2016, p. 90).

while for traditional quotations it was 4.27%, confirming BTC's higher risk as an investment. The highest daily increase and decline against the opening price occurred for the BTC/USD pair – the rates of return were 97.96% and –56.57%, respectively.

The biggest changes to quotes of traditional instruments took place with the price of gold and euro (XAU/EUR). However, the scale of changes was many times lower than that afflicting the BTC/USD pair – the highest and the lowest value shift were, respectively, 14.19% and –9.60%. The average intraday range (in relation to opening price) for BTC quotes was 59.36% while for the rest of the financial instruments in this study, it was 8.48% (Bala, Kopyściański & Srokosz 2016, p. 89–91; Kądziołka 2016, p. 12–14, 38–42; <https://coinmarketcap.com/#EUR>, accessed: 15.08.2017).

In order to assess the full impact of BTC's volatility on financial markets, a daily correlation of return rates has been made (Table 2). The correlation between the rate of BTC's return and national currencies was positive, while for XAU/BTC quotes, there was a very strong but negative correlation. In turn, the interconnectedness between pairs of traditional financial instruments was weak. Analysis of the daily rate of return on BTC and traditional financial instruments indicates a lack of correlation between tested variables. On the assumption that spurious regression does not occur, it can be concluded that the high volatility of BTC and others cryptocurrencies will not affect the rest of the financial market (Bala, Kopyściański & Srokosz 2016, p. 90–91; Kądziołka 2016, p. 42).

BTC quotes against other financial instruments prove more volatile because, among other things, the BTC exchange rate is determined on market terms, which is neither regulated nor controlled by any supervisory authority. Hence there is no mechanism to prevent currency speculation or limit foreign-exchange risk and potential loss. The high volatility of BTC quotes is also a result of low market turnover in contradistinction to other instruments. To sum up, all of these factors limit the trust people are willing to put not only in BTC, but all cryptocurrencies, regardless of whether they were created to perform payment functions. With such a limited trust in reality, cryptocurrencies are treated not as a currency but a high-risk financial asset (Bala, Kopyściański & Srokosz 2016, p. 89, 91; Markiewicz & Nowak 2015, p. 91; Homa 2015, p. 134).

## **8. The Development of Cryptocurrencies and Their Impact on the Economy**

The increased use and distribution of cryptographic currencies can have specific effects on the the economy, particularly in public finance, the banking system, financial markets, and even for individual users. Cryptocurrencies can

significantly expand the grey economy, creating as they do the opportunity to establish and operate an unregistered business in which deals (e.g. for drugs and crimes) are settled by unidentified entities. They are also used to launder money. The anonymity of users, speed of execution of transactions and the ability to exchange them on stock exchanges makes it possible to introduce laundered money into an economy or to finance illegal activities. The common denominator of these threats is mainly users' anonymity, which contributes to reducing the effectiveness of fiscal control by authorised state agencies, potentially reducing public finance sector revenue. It is also unfair to those who square up to their tax liabilities.

For the banking sector, cryptocurrencies are perceived as competition and a threat to credit institutions, and as a result also to deposit money that they create in the process of multiplying deposits. Replacing national currencies created by central banks with cryptographic currencies would mean that both central banks (which would lose control of money supply) and commercial banks would no longer be needed. Banks currently have a monopoly on creating money, which may be one factor that prevents cryptocurrencies – the supply of which is limited – from developing further. For reference, there are 21 million units of Bitcoin available. While the inability to create new units reduces the risk of inflation, affecting economic stability, limited supply, can in the long term, on the other hand, promote deflation. Furthermore, as the number of services and goods increases relative to the constant quantity of cryptographic currency units, a shortage of the latter would ensue. Cryptocurrencies would therefore cease to fulfill a medium of exchange function in favour of the store of value function (further explaining why cryptocurrencies are not able to function as basic money). The idea of replacing national currencies with cryptographic currencies would undoubtedly bring economic growth to a standstill (Bala, Kopyściński & Srokosz 2016, p. 102–105; Kądziołka 2016, p. 35–37; Markiewicz & Nowak 2015, p. 91; Homa 2015, p. 21).

Cryptocurrencies ensure a higher level of privacy than standard electronic payments. For this reason, they have been repeatedly used to regulate transactions for illicit sales of weapons or drugs, ransom demands, and to finance terrorism (e.g. the Silk Road service closed by the FBI in October 2013). There are also the multiple controversies their numerous similarities to pyramid schemes have given rise to, foremost among them the “network effect”. This phenomenon is based on the fact that economic benefits for a current group of cryptocurrency holders increase as new users get in on the action. This applies especially to the creators and users who have been invested from the beginning, and who have an advantage over the rest due to their ability to mine cryptocurrencies in the early stages at minimal cost. With time, a larger number of individuals cause the use and distribution of cryptocurrencies to rise, pushing up their market value and



providing an opportunity to exchange them for national currencies or other real goods and services. Other similarities include the fact that new users exchange on cryptographic currency stock exchange (or currency exchanges) official and legally-accepted national currencies for virtual money which has no intrinsic value. An excellent example of a pyramid scheme is a trading platform called cryptodouble.com. The whole mechanism of this platform was based on promise of doubling deposits of each user after they have 100 hours of transactions under their belt. At the beginning, payments were made from contributions from new users. However, after some time, the stock exchange was shut down. As a result, users irretrievably lost 2233 Bitcoins, or roughly 9,400,930 USD.

In summary, a new type of pyramid scheme, one based on cryptocurrencies, has emerged. A crucial characteristic is the significant imbalance in the number of BTC holders. As Table 3 shows, 0.78% of users own and control 89.20% of the total BTC market value. This can lead a minority of BTC holders to profit thanks to their dominance in the market. Moreover, BTC's total market capitalisation is approximately 67 trillion USD, a number greater than almost 85% of the listed companies in the S&P 500. The manic sentiment and price activity is very similar to the tulip bubble of the 17th century in the Netherlands (Bala, Kopyściański & Srokosz 2016, p. 91–95; Szymankiewicz 2014, p. 87–90, 92–95; Kądziołka 2016, p. 37; <https://www.cryptocoinsnews.com/bitcoin-ponzi-cryptodouble-disappears-least-2233-bitcoins/>; <https://www.theguardian.com/technology/2013/oct/08/silk-road-hack-suspicion-fbi-server>; <https://www.cnbc.com/2017/07/20/bitcoin-bubble-dwarfs-tulip-mania-from-400-years-ago-elliott-wave.html>, accessed: 15.08.2017).

Table 3. Distribution of BTC

Balance	Number of Addresses	Number of Addresses in Total (%)	Number of BTC Coins	Value in USD
0–0.001	11,317,126	59.4	2,078	8,460,766
0.001–0.01	3,302,678	17.4	12,563	51,143,169
0.01–0.1	2,588,875	13.6	80,872	329,225,318
0.1–1	1,211,173	6.4	396,687	1,614,892,334
1–10	472,576	2.5	1,291,276	5,256,711,999
10–100	129,798	0.7	4,340,986	17,671,912,361
100–1,000	16,497	0.1	3,828,754	15,586,641,907
1,000–10,000	1,672	0.0	3,546,960	14,439,474,299
10,000–100,000	118	0.0	2,898,511	11,799,677,004
100,000–1,000,000	1	0.0	119,080	484,766,441
Aggregated	19,040,514	100	16,517,767	67,242,905,598

Source: <https://bitinfocharts.com/pl/top-100-richest-bitcoin-addresses.html> (accessed: 21.08.2017).

Table 4. Advantages and Disadvantages of Use Cryptographic Currencies by Individual Users

Advantages	Disadvantages
<p>Anonymity – currently, all transactions executed through banks are recorded and subsequently used by enterprises for sales and marketing purposes. Cryptographic currencies, also called “confidential currency”, ensure privacy of transactions, because it is unknown who is at the given address</p>	<p>Lack of transparency – cryptocurrency stock exchanges are unregulated and have obscure ownership and organisational structure. For these reasons and other numerous scandals associated with cryptographic currencies (e.g. Mt. Gox or cryptodouble.com case), the possibility of using cryptocurrencies for settlement in the future is very questionable</p>
<p>Transaction time – bank transfers are dependent on the time of the outgoing and incoming sessions. In the case of international transfers, the waiting time can be up to three working days. Realising transfers in cryptocurrencies, regardless of geographic reach, lasts up to 12 minutes</p>	<p>High volatility – according to research (Table 1), transaction motives are suppressed by speculation. This has an impact on the very high price volatility of cryptographic currencies (high risk of purchasing power loss). For this reason, not all businesses accept payments in cryptocurrencies, which causes individuals not to be able to use their virtual currency to settle payments</p>
<p>Lower transaction costs – an inseparable elements of the banking system are transaction fees. In the case of cryptographic currencies, fees are minimal or not at all</p>	<p>Anonymity and irretrievability of transactions – incorrect address funds to be transferred to, or loss of control over the wallet, causes irretrievable loss of means</p>
<p>Double spending – relies on the use of same units in many different transactions. Cryptocurrencies eliminate this problem because there is no way to recover units that have been sent</p>	<p>The lack of guarantee – with traditional currencies, customer deposits are guaranteed up to a certain amount. By contrast to aggregated funds in the banking system, cryptocurrencies do not have any formal money-back guarantee</p>
<p>Generating new units – traditional money is created by the banking system. In the case of cryptocurrencies, users create and implement new entities. However, the more units that are dug up, the harder it becomes to get new ones</p>	<p>Cyber-attacks – both credit institutions and cryptographic currency users are vulnerable to hacker attacks. However, the difference between them is that in the first case banks are responsible for the security of collected funds, while cryptocurrencies holders do not have any technological support to protect their portfolios. They are responsible for their protection</p>
<p>Diversification – cryptographic currencies can complement stocks, bonds or other financial instruments in an investment portfolio</p>	<p>Virtual character – in contrast to traditional currencies (primarily to banknotes and coins), without access to a computer or smart phone and internet network, use of cryptographic currencies is impossible. Any breakdown of the computer system, power outage or lack of internet access renders users unable to use their funds</p>

Source: the authors, based on (Bala, Kopyściński & Srokosz 2016, p. 99–102; Kądziołka 2016, p. 34–35; Markiewicz & Nowak 2015, p. 180–184; Szymankiewicz 2014, p. 81–90; Prabucki 2014, p. 71–72).

Acceptance of any object or secure verifiable record as money is an economic and legal issue. From an economic point of view, money is a unit of account, a medium of exchange, a standard of deferred payment and a store of value. Cryptocurrencies function indirectly as a unit of account. Although it is possible to compare the value of goods and services expressed in BTC or other cryptographic currencies, due to the limited possibilities for exchanging most goods and services for cryptocurrencies, determination of their value occurs only after conversion from a national currency. Because the value of cryptographic currencies is influenced by the exchange rate of national currencies (which have a specified purchasing power), and cryptocurrencies are treated as investment assets, it is impossible to directly determine prices in particular cryptographic currencies. The next function (medium of exchange) is related to use of money (as a universal equivalent) to facilitate buy-sell transactions for goods and services. However, it is important to keep in mind that intermediary instruments must be accepted by both parties to a transaction. This function is not fulfilled by cryptographic currencies, because they are not widely accepted intermediary instruments in routing. The third function of money – a standard of deferred payment, which refers to the possibility of regulating different types of liabilities (e.g. tax, loans, social contributions, salaries, etc.) – is also not achieved by cryptocurrencies.

This is so because, first, cryptographic currencies are not regulated by law, which is why it is impossible to use them to regulate tax liabilities. Second, because it is impossible to control their quantities in the economy, they are not approved by the monetary authorities to be used as money in circulation. Finally, in terms of their functioning as a store of value, the limited supply of cryptocurrencies, in accordance with theory, should cause them to appreciate in the long run, which may induce people to accumulate them. On the other hand, the intangible form of cryptographic currencies (as opposed to gold, for example), the lack of guarantee of their value and state supervision (lack of guaranteed funds, as in the case of bank deposits) all combine to reduce people's confidence in accumulating savings using cryptocurrency as the medium. To sum up, cryptocurrencies do not fulfill the functions of money. In this light, they are treated more as financial instruments (investment assets) than as money (Bala, Kopyściański & Srokosz 2016, p. 95–98; Owsiak 2015, p. 128–129; Prabucki 2014, p. 80).

Currently, the total number of Bitcoin users is over 19 million. Benefits and threats of use cryptographic currencies by individual users are presented in Table 4.

## 9. Conclusion

Mining (digging up) new cryptographic currency units requires high computing power and a lot of time, which drives up electricity consumption. Nevertheless, more serious issues than the technical aspects of growth in use and distribution of cryptocurrencies are related to economic aspects. Cryptographic currency quotations show that these currencies are far more volatile than traditional national currencies and other financial instruments, limiting people's trust. It should also be emphasised that the original premise of cryptocurrencies as a vehicle for transactions gave way to speculation, so investment assets are treated as a cryptographic currency.

Secondly, how cryptographic currencies are created and distributed qualifies them as a potential new type of pyramid scheme. The creators of cryptocurrencies, who produce at low cost and quickly obtain new units, have the most to gain. When cryptographic currencies achieve a high value, units are sold for traditional currencies, which can cause quotations to collapse and spell tremendous losses for other users. Third, the anonymity of cryptocurrency users has far-reaching consequences, giving rise to money laundering, financing illegal activities (including terrorism), and harming public finances (i.e. taxes are evaded). Finally, in view of current economic theory, cryptographic currencies cannot be considered money because they do not fully achieve all of the functions of money. The unit of account function is performed indirectly, while they are entirely unable to function as a medium of exchange and standard of deferred payment, as it is debatable whether they store value. Even if cryptocurrencies fulfill these functions in the future and become fully legal tender, deflation will result in the long term.

In the longer term, further development and distribution of cryptographic currencies will depend primarily on the legal regulations and public oversight of entities that use cryptographic currencies (including the creation of a prevention mechanism in order to counteract the formation of pyramid schemes). For cryptocurrencies to completely replace current national currencies, how the economy functioned would have to change entirely.

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## **Rozwój i wpływ walut kryptograficznych na stabilność i funkcjonowanie systemu finansowego – wybrane aspekty**

(Streszczenie)

Pierwsza kryptowaluta (bitcoin) powstała w 2009 r. Od tego czasu kryptowaluty bardzo dynamicznie się rozwijają, systematycznie wzrasta też liczba ich użytkowników. Celem artykułu jest analiza i ocena rozwoju i wzrostu częstości wykorzystywania walut kryptograficznych oraz ich wpływu na stabilność i funkcjonowanie systemu finansowego. Rozważano też, czy kryptowaluty spełniają funkcje pieniądza. Poruszono kwestie techniczne oraz prawne walut kryptograficznych, dokonano analizy statystycznej – notowania bitcoina zostały porównane do notowań innych tradycyjnych instrumentów finansowych z wykorzystaniem stopy zwrotu, współczynnika zmienności, rozstępu oraz współczynnika korelacji Pearsona, oraz oceny wpływu na finanse publiczne, sektor bankowy czy indywidualnych użytkowników. Przeprowadzone analizy pozwalają sformułować wniosek, że waluty kryptograficzne nie mogą być traktowane na równi z pieniędzmi (stanowią one aktywa inwestycyjne), ich notowania wykazują bardzo dużą zmienność względem innych tradycyjnych instrumentów oraz występuje bardzo duże podobieństwo walut kryptograficznych do piramid finansowych.

**Słowa kluczowe:** waluty kryptograficzne, bitcoin, stabilność, system finansowy.

| *Alina Klonowska*

# | The Impact of European Funds on Public Expenditure in Poland

## Abstract

The main purpose of the study is to evaluate the impact of European funds on the level and structure of public expenditure in Poland. Basically, the research period covers the years 2000–2016. In pursuit of objectivity, the following hypothesis is put forth: EU funding has a significant impact on the amount and structure of public expenditure.

The analysis of data on the amount of European funds supporting the state budget in Poland proves that the share of these funds in the financing of public tasks is high and the structure of public expenditures is significantly changed. In the period 2010–2015 these funds accounted for about 88% of public expenditure. It is clear that they have systematically, if gradually, risen. Over the period 2014–2015, funding exceeded 90% of total expenditure.

**Keywords:** European funds, operational programme, the EU funds budget, public expenditure.

**JEL Classification:** H50, H60.

## 1. Introduction

Poland's long presence in the EU has yielded measurable benefits. They are evidenced not only by the results of analyses on the development of macroeconomic indicators, but above all by public opinion. The latest data show that for over two years the percentage of supporters of Poland's membership in the European Community has not fallen below 80%. According to CBOS, the majority

are supporters in all socio-demographic groups (*Polska w Unii Europejskiej...* 2016, p. 2). Furthermore, the Supreme Chamber of Control (NIK), referring to estimates by the Ministry of Infrastructure and Development, notes that around 20% of Poland's average annual GDP growth is the result of the implementation of investments co-financed by EU funds (*Analiza...* 2014, p. 198). Public expenditure is one channel by which EU funds are transmitted to the country's economy.

The main purpose of the study is to assess the impact of European funds on the level and structure of public expenditure in Poland. The study answers the following research questions: Did the EU funds impact the amount and structure of public expenditures in Poland? What is the direction of changes in funding by EU funds? Can the changes to public expenditures be assessed as beneficial? The subject of the study is public expenditure in Poland in the period 2000–2016. Its main hypothesis is that EU funding has a significant and positive impact on the amount and structure of public expenditure in Poland.

The paper is organised as follows. Chapter one presents a review of the literature. The second chapter outlines the concept and structure of funding for EU policies based on financial perspectives. This chapter also shows the financial flows from the EU budget funds received by Poland.

The third chapter discusses the directions for implementation of national public policies that reflect expenditure for the implementation of operational programmes. The fourth chapter presents an analysis of expenditure by economic groups, including expenditure on financing and co-financing of projects involving EU funds.

## 2. Literature Review

In the subject literature, the factors that determine the level of public expenditures are interpreted in a variety of ways. It is noteworthy that the level and structure of public expenditures are influenced by historical conditions as well as the socio-economic doctrine taken by the public authorities (Owsiak 2017). A. Kasperowicz-Stępień (2011) sees both the level and structure of expenditures as a problem, the solution to which is often a compromise between different options and arguments. A. Możdziej (2009), in turn, emphasises systemic changes. Looking at the structure of expenditures, the author argues that the change in the rules of recognising expenditures for the implementation of programmes and projects within the structural funds and in-community initiatives (which were included in the state budget in 2006) caused an increase in legally determined expenditures. As of 2010, the budget of European funds has been excluded from the state budget. In accordance with the economic approach of expenditures, the

state budget included the contribution to the EU budget as well as the amounts related to the co-financing of EU projects. Such an approach has left some with an opinion of EU funds, which tighten up state budgets with their already high share of legally determined expenditures.

At the same time, EU funds are considered crucial to the economic, social and administrative development of Central and Eastern European countries (Klimowicz 2010). The use of European funds brings about numerous benefits, which are viewed as incentives to making new investments in both the public and private sectors. M. Tomova *et al.* (2013) showed that European funds help Member States improve their socio-economic development, particularly when it is linked with solid domestic fiscal and macroeconomic policies. P. Wostner and S. Šlander (2009) confirmed that cohesion policy effectively increases Member States' structural expenditures, fulfilling one of the conditions essential for the effectiveness of EU transfers. O. Podlińska and J. Gołębiwski (2013) also conducted research concerning the impact of European funds on the expenditure level. Their research examined the impact of EU support schemes on municipal investment expenditures, and established that investment expenditures increased after Poland's accession to the EU. Their analysis of the share of EU funds in investment expenditures indicates that the municipalities showed greater activity and effectiveness in obtaining EU funds in the first years of membership. However, A. P. Czepiel (2016) has emphasised that while the efficiency of spending funds is thought important, little is known about it. The author suggests that the reason for this is the underdeveloped assessment systems indicating the impact of European funds – at national, local and regional levels in Poland – on economic growth.

To the best of my knowledge, empirical studies on the influence UE funds have on the extent and structure of public expenditure according to economic groups in Poland is poor. This study addresses both of these areas. The subject literature has most often dealt with public finances in general (Sawulski 2016), the possibilities of financing investments within particular EU programmes, the influence of EU funds on employment, economic growth and the development of the country and its regions (Sosińska-Wit 2014). The literature also illustrates the difficulties that exist in measuring the efficiency and effectiveness of public spending (Mandl, Dierx & Ilzkovitz 2008).

### **3. The Limits and Directions of Funding Defined by the Multi-annual Financial Framework**

Data from the European Commission ([www.ec.europa.eu](http://www.ec.europa.eu), accessed: 25.01.2017) show that expenditure on implementing the common agricultural



policy was predominant in the structure of expenditure of the EU general budget between the years 2000–2006 (48% on average per annum). These were, among others, expenditure on direct subsidies for agricultural production and subsidies for exports of agricultural products to third countries. The second most significant group involved structural funds (31% on average). 2007 marks the introduction of a new breakdown of expenditures adjusted to the priorities defined for this period. The new main lines of financing are to provide sustained economic growth, with over 41% of total funds being allocated annually and natural resources management (45%). It is important that, over the period 2007–2011, expenditure on the conservation of natural resources, i.e. agriculture, rural development, environment and fisheries, outweighed the costs of sustainable economic growth. It was only in 2012–2013 that the relationship changed in favour of economic expenditures. In the financial perspective for 2014–2020, the structure of the financial framework is similar. There are still two main areas to be financed, i.e. economic development and natural resources. Over 47% of the annual average is transferred under EU funds for economic purposes, and in the latter case it is close to 40% ([www.ec.europa.eu/budget](http://www.ec.europa.eu/budget), accessed: 25.01.2017, *Multiannual Financial Framework... 2013*, p. 8). Table 1 presents details of the main EU funding areas.

Table 1. EU Funding Areas for 2000–2015 (Billion EUR)

Specification	2000	2001	2002	2003	2004	2005	2006
Common agricultural policy	40.5	41.5	43.5	44.4	43.6	48.5	49.8
Structural activities	27.6	22.5	23.5	28.5	34.2	32.8	32.4
Specification	2007	2008	2009	2010	2011	2012	2013
Sustainable development, including: competitiveness and cohesion for growth and employment	43.7	45.6	44.7	48.8	54.7	61.6	71.3
Natural resources protection and management	54.6	54.8	55.9	56.6	57.4	59.1	59.5
Specification				2014		2015	
Intelligent growth supporting social inclusion, e.g.: competitiveness for growth and employment, and social and territorial economic cohesion				67.7		68.0	
Sustainable growth: natural resources				56.6		58.1	

Source: the author's own elaboration based on: [www.ec.europa.eu/budget/figures/interactive/index\\_en.cfm](http://www.ec.europa.eu/budget/figures/interactive/index_en.cfm) (accessed: 25.01.2017).

Between 2000 and 2006, EU funding was focused on the common agricultural policy, with an average annual budget of around 61%. The pace of changes in the amount of expenditure disbursed in both areas was similar and amounted

to approx. 3%. In turn, for most of the period 2007–2013, EU funds were mainly directed towards conservation and management of natural resources. The appropriations accounted for 54%. It was only in 2012–2013 that higher amounts were earmarked for sustainable economic development. In the whole period, on an average annual basis, they grew at a rate of about 8%. This shows that since 2007, despite the lower amounts spent on sustainable development, their rate of change demonstrated the growing importance of financing this area. This trend continued until 2015, when the money earmarked for smart growth was around 70 billion EUR.

So far, Poland has benefited from three financial perspectives (2004–2006, 2007–2013 and 2014–2020). The country has been the biggest beneficiary of cohesion policy and the Rural Development Programme and the main beneficiary of Direct Subsidies under the Common Agricultural Policy among the new EU Member States. Poland first contributed to the EU budget for a full year in 2005. Its contribution was 2.4 billion EUR (*Analiza...* 2006). In the following years, the amount rose systematically, and after ten years the contribution has increased by 75%. Since 2004, over 40 billion EUR has been transferred to the EU budget and 146 billion EUR has been returned.

Funds raised by 2016 totalled 130.5 billion EUR, nearly quadrupling the total contributions paid into the EU budget. On a net basis, i.e. after deducting contributions and refunds, the amount obtained was over 87 billion EUR for the entire period.

Over the last dozen years, there has been an increase in transfers from the EU from 2.5 billion EUR in 2004 to over 17 billion EUR in 2014 (without deduction of contributions and returns). Despite this, the EU funds accounted for 1.2% of GDP in 2004 and 3% in 2015. Net funding received by Poland from the EU budget, after deduction of contributions and refunds, accounted for 0.57% in 2004, while in 2015 it was 2% of GDP (calculations based on data from: [www.stat.gov.pl](http://www.stat.gov.pl), [www.nbp.pl](http://www.nbp.pl), [www.nik.gov.pl](http://www.nik.gov.pl), accessed: 12.07.2017).

In the period 2000–2016 the largest transfer of funds – 47 billion EUR over the period, accounting for 36% of the total gross proceeds – from the EU budget was received for the Structural Funds. The Cohesion Fund was the second largest fund. The funds raised amounted to more than 26 billion EUR (20%), approximately 19% of which was allocated for direct payments, while the sum of over 16 billion EUR (12%) was allocated to the Rural Development Programme. Although the financial perspective for 2014–2020 changed the amount of funds so that they were reduced, this did not negatively impact the amount of funds allocated for Poland. In fact, they increased from 101.5 billion EUR in 2007 to 105.8 billion EUR in 2013. Funds for cohesion policy for Poland increased from 69 billion EUR to 72.9 billion EUR. Finally, the Common Agricultural Policy was allocated

1.6 billion EUR more ([www.msz.gov.pl/pl/polityka\\_zagraniczna/zagraniczna-polityka\\_ekonomiczna/wieloletnie\\_ramy\\_finansowe](http://www.msz.gov.pl/pl/polityka_zagraniczna/zagraniczna-polityka_ekonomiczna/wieloletnie_ramy_finansowe), accessed: 25.01.2017)<sup>1</sup>.

Table 2. Financial Flows in 2004–2015 (Billion PLN)\*

Specification	Transfers from EU	Membership Contribution**	Balance of Settlements of Poland with EU
2004	12.5	5.8	6.7***
2005	16.1	9.9	6.2
2006	20.5	9.9	10.6
2007	28.1	10.8	17.3
2008	26.9	12.2	14.7
2009	40.2****	13.4	26.7
2010	44.6	14.3	30.3
2011	58.5	14.9	43.6
2012	64.6	15.9	48.7
2013	65.6	18.1	47.5
2014	71.7	17.2	54.4
2015	54.6	18.2	36.4

\* According to the average exchange rate of the National Bank of Poland at the end of the year.

\*\* Including the return of funds to the EU budget.

\*\*\* The net gain was due to, inter alia, the impact of pre-accession funds and advances for the implementation of programmes co-financed by the Structural Funds.

\*\*\*\* Without funds received under the Financial Mechanisms (Norwegian MF and MFI) and the Swiss-Polish Cooperation Programme.

Source: the author's own study based on: <http://www.mf.gov.pl/ministerstwo-finansow/dzialalnosc/unia-europejska/transfery-finansowe-polska-ue> (accessed: 20.12.2017).

According to the new financial framework, the largest part of the allocations comes from cohesion policy, Common Agricultural Policy and Common Fisheries Policy. Additional funds come from programmes managed directly by the European Commission. The Partnership Agreement shows that the total allocation for Poland before transfers was 82.5 billion EUR (at current prices). Mandatory transfers to instruments and programmes managed directly by the European Commission were included, e.g. Fund for European Aid to the most Deprived – FEAD), over 475 million EUR (*Programming...* 2015, p. 158). It is worth noting that the voivodship self-governments will manage a larger amount of EU funds than is currently the case. Between 2007 and 2013, they accounted for 25% of all funds available to Poland, while in the new perspective it will be almost 40%

<sup>1</sup> In order to compare the multiannual financial framework for 2014–2020 with the previous perspective, the results of the negotiations are presented in 2011 constant prices.

(*Gospodarcze i społeczne...* 2015, p. 10). It can therefore be supposed that this will bring real benefits in the form of better use of EU funds in future perspectives since local authorities better know the needs of local society. Furthermore, new perspectives increase the speed of development of Poland's regions and reduce disparities in GDP *per capita* between these regions.

Table 2 presents data on the amount of transfers and membership fees converted into national currency.

In 2004–2015, the total amount of funds transferred from the EU to Poland amounted to over 500 billion PLN, of which 160 billion PLN was the fee paid. The decrease in contributions to the European Union budget in 2014 resulted from corrections to the general budget made by the Council and the European Parliament during the financial year (*Analiza...* 2010, p. 71). As a result, the balance of Poland's settlements with the EU oscillated around 300 billion PLN in the period.

#### **4. Use of European Funds under Operational Programmes**

The directions of implementation of national public policy are laid out in the National Development Plans that cover the years of the next financial perspectives. Implementation of the National Development Plan is provided by operational programmes financed from national funds or co-financed from community funds (Ustawa z dnia 20 kwietnia 2004 r. ... 2004). The programmes can be national, regional (covering the voivodship) and sectoral. Each programme determines the type of planned projects for which financial support is provided. As in the 2007–2013 perspective, allocations for 2014–2020 will be based on regional operational programmes implementing national operational programmes. They are intended to support the development of all regions. In this case, the funds are managed not by central authorities, but self-government institutions. Unlike in the previous perspective, regional programmes are dual-fund, meaning that they are financed by the European Regional Development Fund and the European Social Fund. The regional programmes will have a much higher share of the funds allocated to Poland than in previous years. For the first time, voivodships will manage funds under the European Social Fund. With this solution, local governments will have greater autonomy in achieving their development goals.

Reports from the Supreme Chamber of Control show that in the following periods, the following expenses were completed:

- 1) in the years 2000–2004, 8.4 billion PLN was spent. Funds came from EU pre-accession assistance programmes and other non-recoverable foreign sources, and from 2004 also from the Structural Funds, the Cohesion and the Common

Agricultural Policy Fund (own calculations, *Sprawozdanie... 2001–2016*). At the beginning of the implementation of the programmes, the use of funds was very low. NIK data also shows that one of the reasons for the low absorption of Structural Funds involved the complex management and control system, which routinely imposed on individuals the need to carry out tasks not required by community law, and even duplicated some of the actions at subsequent levels of the system (*Analiza... 2009*, p. 167);

2) between 2005 and 2007, over 53 billion PLN was used. As before, financial resources came from the same sources of funding (*Analiza... 2006–2008*);

3) in the years 2008–2009, projects were implemented for (*Analiza... 2009–2010*):

– National Development Plan (NDP) 2004–2006 – 60.8 billion PLN,

– National Strategic Reference Framework (NSRF) 2007–2013 – just over 21 billion PLN,

– Common Agricultural and Fisheries Policy – about 3 billion PLN,

– other non-recoverable funds from the European Union (pre-accession assistance, transitional measures and the Norwegian Financial Mechanism, EEA MF, as well as the Swiss-Polish Cooperation Programme and other sources) – 2,3 billion PLN.

Expenditures in this period totalled 87.1 billion PLN.

In the years 2010–2015, the amounts spent were significantly higher, totalling over 385 billion PLN. Table 3 provides detailed data.

In order to implement the assumptions of the European Union policy, much lower amounts were spent from the state budget. In the period under review, the expenditures exceeded just 50 billion PLN. Table 4 presents the data.

The data in Tables 3 and 4 show that expenditures of 435.7 billion PLN were used in 2010–2015, or around 73 billion PLN annually. Less than 12% of the funds came from the state budget, while 88% of the expenditure was financed from EU budget funds. At the same time, there was a systematic and gradual increase in financing of expenditure with the funds from the EU budget. Over the period 2014–2015, this funding exceeded 90% of total expenditure. It should be noted that the actual implementation of income depends on the level of completed expenditure, which is affected by the pace of implementation of individual projects. The level of absorption of funds and the pace of programme implementation depends on many factors, e.g. changes in project implementation schedule and prolonged tendering procedures, which often push projects into the next financial year. As a result, smaller resources launched for the implementation of projects financed under the budget of European funds had an impact on the lower level of expenditure under the budget of the European funds, which in turn translated into lower execution of revenues in relation to the planned budget.

Table 3. Expenditures Made from the EU Budget in 2010–2015 (Billion PLN)

Operational Programme (OP)	2010	2011	2012	2013	2014	2015
OP Infrastructure and Environment	8.47	16.85	25.65	17.07	20.84	19.19
OP Innovative Economy	3.87	4.89	5.37	6.93	7.24	6.66
OP Human Resources Development	7.14	7.11	6.42	6.92	6.93	3.07
OP Technical Assistance 2007–2013	0.25	0.27	0.26	0.28	0.39	0.27
OP Development of Eastern Poland	1.16	1.55	1.26	1.41	1.99	2.02
OP of the European Territorial Cooperation	0.03	0.02	0.01	0.02	0.01	0.01
European Neighbourhood and Partnership Instrument	0.01	0.01	0.04	0.04	0.02	0.02
Regional Operational Programmes	13.53	12.44	10.90	9.30	9.16	9.14
ESF grants	0.00	0.00	0.00	0.00	0.00	0.00
Total NCS 2007–2013	34.45	43.14	49.92	41.98	46.57	40.38
Swiss-Polish Cooperation Programme	0.07	0.03	0.19	0.19	0.29	0.28
Norwegian Financial Mechanism and EEA FM 2009–2014	–	0.00	0.01	0.09	0.31	0.68
OP Sustainable Development of the Fisheries Sector and Coastal Fisheries	0.32	0.62	0.63	0.60	0.55	0.27
Common Agricultural Policy	14.42	18.24	18.85	22.33	22.06	25.19
Financial Perspective 2014–2020	–	0.00	0.00	0.00	0.00	2.42
Total EU budget	49.27	62.02	69.59	65.20	69.78	69.22

Source: (*Sprawozdanie... 2011–2016*).

Table 4. Expenditures from the State Budget as Part of Financing and National Co-financing in 2010–2015 (Million PLN)

Operational Programme (OP)	2010	2011	2012	2013	2014	2015
OP Infrastructure and Environment	437.8	550.7	1,207.3	716.0	306.9	1,288.9
OP Innovative Economy	556.2	948.4	930.0	1,048.5	1,276.7	1,263.4
OP Human Resources Development	756.7	848.9	793.6	828.8	847.0	428.1
OP Technical Assistance 2007–2013	48.5	51.1	49.8	51.7	71.9	47.4
OP Development of Eastern Poland	159.3	87.8	125.0	85.5	90.4	219.2
OP of European Territorial Cooperation	10.2	12.5	10.7	11.0	9.6	9.1
European Neighbourhood and Partnership Instrument	0.0	0.0	4.4	4.1	0.9	1.2
Regional Operational Programmes	647.6	659.4	778.0	635.3	679.5	699.4
ESF grants	–	–	0.0	0.0	0.0	0.0
Total NCS 2007–2013	2,616.3	3,158.7	3,898.7	3,381.0	3,282.8	3,956.9
Swiss-Polish Cooperation Programme	0.6	2.4	10.3	16.0	17.1	9.5
Norwegian Financial Mechanism and EEA FM 2009–2014	–	–	0.6	4.1	30.3	65.2
OP Sustainable Development of the Fisheries Sector and Coastal Fisheries	108.5	204.8	188.5	213.5	185.9	94.1
Common Agricultural Policy	6,107.4	6,152.9	5,151.5	5,164.9	3,696.9	2,745.5
Financial Perspective 2014–2020	–	–	–	–	22.6	101.0
Total state budget	8,832.8	9,518.8	9,249.6	8,779.5	7,235.6	6,972.2

Source: (*Sprawozdanie... 2011–2016*).

## 5. Allocation of Funds by Economic Group Expenditure

A significant share of expenditure related to the co-financing of EU projects can be noted on the expenditure side of the state budget. National co-financing aims to provide the financial resources needed to implement EU projects. The unit implementing the project requires temporary financing of part of the expenditure. EU funding is provided through reimbursement of part of the expenditure previously made. Table 5 shows state budget expenditure by economic group, including expenditure for co-financing implemented in 2006–2015. The analysis period was shortened accordingly as the first pre-financing expenditure for EU projects from the state budget was incurred in 2006.

The data presented in Table 5 shows that throughout the period, expenditures made from the state budget in the framework of pre-financing and co-financing systematically increased. The share of these expenditures in the overall amount fluctuated on average by around 5% over the entire period. In 2007–2009, however, this relation increased significantly, and compared to the other years it was considerably higher. Unlike in the past, spending in 2010 with EU funds decreased by 7.2 percentage points from the previous year. This was due to changes resulting from the separation of the state budget of revenue and expenditure and the deficit of EU funds. The name of this pre-financing spending group was also changed into co-financing projects funded by the European Union, as almost all of the financing of expenditure that is reimbursed from the EU was transferred to the budget of European funds. The budget of national funds covered the expenditure on co-financing of projects implemented with European funds, financing of technical assistance projects, projects funded by EFTA member states, and expenditure on projects involving other EU budget funds not included in the budget of European funds (*Sprawozdanie...* 2010).

The separate treatment of expenditure of “financing of projects involving EU funds” in the group of economic expenditures has made it difficult to figure out, for example, what capital expenses were actually incurred, and the value of grants and subsidies cannot be reliably determined. Due to the differing data layout and their lack of comparability, it is possible to present allocations for co-financing, after inclusion in specific categories of economic expenditure, for the 2008–2012 period alone. Table 6 presents the data.

Expenditure for co-financing projects involving EU funds is the category that actually includes spending belonging to the above groups of economic expenditures. Comparison of expenditures presented in Table 6 with data in Table 5 shows that total expenditure has not changed. Both the cost of servicing the public debt and the contribution to the EU budget have remained unchanged. Other categories have changed. Throughout the period, the amount of grants and



Table 5. Expenditure of the State Budget by Economic Groups in 2006–2015 (Billion PLN)

Specification	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Grants and subsidies	108.00	113.68	130.79	133.95	145.24	148.46	155.36	155.20	152.26	169.05
Benefits to individuals	17.92	22.99	21.17	20.11	20.89	21.88	23.00	23.95	24.43	24.65
Current expenditure of budgetary units	41.48	45.49	48.51	51.23	53.91	55.34	57.23	58.42	60.03	61.94
Capital expenditure	9.80	13.26	14.92	14.65	15.44	14.92	13.57	12.88	15.25	20.06
Settlements with banks	1.01	1.19	–	–	–	–	–	–	–	–
State Treasury debt servicing	27.77	27.50	25.12	32.23	34.14	35.96	42.11	42.46	34.46	29.17
EU own resources	9.84	10.63	12.22	13.42	14.35	14.73	15.94	12.83	17.26	18.20
Co-financing of projects involving EU funds	6.88	17.58	25.17	32.43	10.92	11.40	10.80	10.30	8.83	8.68
Total	222.70	252.32	277.89	298.03	294.89	302.68	318.00	321.35	312.52	331.74
Share of expenditure on co-financing EU projects in total state budget expenditure (%)	3.1	7.0	9.1	10.9	3.7	3.8	3.4	3.2	2.8	2.6

Source: (*Sprawozdanie... 2007–2016*).

subsidies increased by 61.6 billion PLN (growth of 8.6%). Subsequently, capital expenditure increased, which in the whole period changed by 24.6 billion PLN (33.4%). Slight changes can also be observed in the current expenditure categories of budget units. In this case the growth was only 1.7%. In 2009, expenditure on co-financing increased by 77%. In the same period, expenditure on grants and subsidies increased by only 15%.

Table 6. Expenditure of the State Budget after Inclusion of Expenditure from the Co-financing of Projects Involving EU Funds in 2008–2012 (Billion PLN)

Specification	2008	2009	2010	2011	2012
Grants and subsidies	148.06	154.13	153.43	157.04	162.78
Benefits to individuals	21.17	20.11	20.89	21.88	23.00
Current expenditure of budgetary units	49.44	52.19	54.85	56.27	57.97
Capital expenditure	21.88	25.95	17.23	16.81	16.21
State Treasury debt servicing	25.12	32.23	34.14	35.96	42.11
EU own resources	12.22	13.42	14.35	14.73	15.94
Total	277.89	298.03	294.89	302.68	318.00

Source: (*Sprawozdanie...* 2009–2013).

The great importance of financing from European funds is primarily reflected in the data on the development of economic expenditures from the state budget and the budget of European funds. A detailed account of this is available from 2007 when the EU funds were included in the budget. The implementation of expenditure in particular economic groups is presented, taking into account the expenditure of the co-financing of projects with the participation of European Union funds group. The data is shown in Table 7.

The level of expenditures both from the state budget and from the budget of European funds is high. A comparison of the expenditures presented in Table 7 with data in Table 6 shows just how crucial the financing expenditure from the budget of European funds is. In the period 2010–2012, the average annual expenditures made exclusively from the state budget and allocated for economic purposes classified by economic groups amounted to 305 billion PLN. Expenditures made from the state budget together with the appropriations from the budget of European funds during the same period amounted to 364 billion PLN. In the years 2010–2012, significant expenditure increases were observed in two groups:

- capital expenditure, 181% on average (expenditure increased by 235% in 2012),
- grants and subsidies, 18%.

Table 7. Expenditures by Economic Groups Made from the State Budget and Budget of European Funds in 2010–2015 (Billion PLN)

Specification	2010	2011	2012	2013	2014	2015
Grants and subsidies (for current tasks)	178.5	187.3	192.6	197.8	193.5	207.3
Benefits to individuals	20.9	21.9	23.0	24.0	24.4	24.7
Current expenditure of budgetary units	55.4	56.8	58.5	59.9	61.4	63.4
Capital expenditure (including grants and subsidies)	39.7	46.8	54.2	42.9	49.9	56.8
State treasury debt servicing	34.1	36.0	42.1	42.5	34.5	29.2
EU own resources	14.3	14.7	15.9	18.1	17.3	18.2
Total	343.0	363.5	386.3	385.2	380.9	399.4

Source: (*Analiza...* 2011–2016).

Total expenditures made from the state budget and the budget of European funds increased by 19%. A significant increase in the size and share of capital expenditures in the total expenditure of the state budget and the budget of European funds is the result of the increasing share of European funds in their financing every year. This data confirm that EU funds have significant influence on the structure of public expenditures in Poland. The direction of changes is beneficial: capital expenditures are growing, which is crucial for the country's development.

In 2013, EU funds accounted for more than half of the budget allocated to finance the investment. In 2012, a record-breaking year in this respect, capital expenditure from the budget of European funds amounted to 37,9 billion PLN, more than double the expenditure of the state budget (*Analiza...* 2012, 2013).

Analysis of state budget expenditure and the budget of European funds shows that these expenditures are predominantly distributed in the form of grants and subsidies for current tasks to other legal entities, both in the public finance sector and in other sectors. The inflow of funds from the EU has significantly increased the importance of capital expenditures, which since 2010 accounted for about 11–12% of total expenditure. Excluding EU funds, capital expenditure accounts for less than 5% of total expenditure. As of 2010, the rate of growth of capital expenditure, together with the appropriations from the budget of European funds, has been on average at almost 9% a year. In the same period, the overall expenditure growth rate was 3% per annum. The increase in both the size and the share of capital expenditure is due to the increasing share of European funds in their financing, which has been increasing each year since 2008.

From 2007 to 2013, total grants and subsidies for financing current and investment tasks increased by 76%, while total expenditure was 53%. During the

period, state budget units spending increased by only 17% (*Sprawozdanie...* 2013, p. 112). 2012 was the investment peak for the financial perspective 2007–2013. In 2013, more funds were invested in investments than in 2010, when the current investment cycle was advanced. For the first time since Poland's accession to the European Union, the share of capital expenditure financed by European funds in total capital expenditure decreased – from 70,1% in 2012 to 65,5% (*Analiza...* 2013, p. 125).

## 6. Conclusion

The implementation of operational programmes within financial perspectives has benefited Poland financially. The data analysis concerning expenditures according to economic groups confirms the significant impact of EU funds on the level and structure of public expenditure in Poland. However, given the systemic changes made in 2007, 2010 and in 2013, it is difficult to obtain uniform information on the amounts used by Poland for the entire research period (2000–2016). We therefore needed to focus on certain periods of analysis. In the period 2000–2009, expenditures used from the EU funds amounted to approximately 87 billion PLN. While in the years 2010–2015, the budget of the European funds financed the implementation of programmes totalling over 385 billion PLN.

In addition to the financial benefits presented, it is important to bear in mind that the EU funding policy or co-financing of EU projects is both encouraging and motivating for undertaking the new initiatives, both public and private. Changes in the level of capital expenditures prove this.

Over the long term, however, it is important what goals will be achieved with these funds. The countries of Central and Eastern Europe largely allocate these funds for road and rail infrastructure. It is also important to focus on research and development, which will support the economy's long-term development.

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## **Wpływ funduszy europejskich na wydatki publiczne w Polsce**

(Streszczenie)

Celem opracowania było dokonanie oceny wpływu środków funduszy europejskich na wysokość i strukturę wydatków publicznych w Polsce. Okres badawczy obejmował lata 2000–2016. Postawiona została następująca hipoteza: środki finansowe pochodzące z funduszy Unii Europejskiej mają istotny wpływ na wysokość i strukturę wydatków publicznych.

Przeprowadzona analiza danych dowiodła, że udział środków z Unii Europejskiej w finansowaniu zadań publicznych jest wysoki, a struktura wydatków publicznych uległa istotnej zmianie. W okresie 2010–2015 środki te stanowiły ok. 88% wydatków publicznych. Zauważyć można także ich systematyczny i stopniowy wzrost. W latach 2014–2015 finansowanie to przekroczyło 90% ogółu zrealizowanych wydatków.

**Słowa kluczowe:** fundusze europejskie, programy operacyjne, budżet środków UE, wydatki publiczne.



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# Producers' Optima in Schumpeterian Evolution\*

## Abstract

The paper presents a production system in the Debreu model of general equilibrium. According to Schumpeter, economic development is possible only on the strength of innovations being introduced. This process provides a sequence of optimal production plans, corresponding to each stage of the innovative evolution. The paper characterises the sequence of optimal plans and provides the conditions for its convergence. Moreover, the limiting production plan is shown to be the producer's optimum in the final state.

**Keywords:** Debreu model, production system, Schumpeterian evolution, innovation, Kuratowski convergence.

**JEL Classification:** C61, C62, D50.

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\* This publication presents the results of a research project financed from a subsidy for the maintenance of research potential, granted to the Faculty of Finance and Law at Cracow University of Economics.



## 1. Introduction

*To the memory of Professor Andrzej Malawski*

The paper examines the production system of the Debreu economy and focus on its Schumpeterian evolution. The goal of a producer is to maximise profit over the set of production. The success of all market agents is possible if and only if there exists a price system common to everybody. In the second part of the paper, the formal description of the production system and the framework of the Debreu model are briefly reviewed.

J. A. Schumpeter (1912) determined two forms of economic life: circular flow and economic development. The first form corresponds to a state when all the processes and agents follow the known economic rules, even if some new good appears in the market. The second form, economic development, can be obtained via creative destruction. We review the basic facts from this theory in the second part of the paper. In the same part we briefly recall some results from the papers of B. Ciałowicz and A. Malawski (2011) and A. Lipieta and A. Malawski (2016), who attempted to describe formally the Schumpeterian mechanisms. This is done to justify the approach in the third part, where we use the fact that in the production system of the Debreu model the Schumpeterian evolution is observed via changing sets of production. However, the above authors' works define the cumulative and innovative extensions of producer's system, requiring in the definitions that the producers must increase their profits. One question remains open here: whether or not a producer who in each stage of the transitions of the economy maximises his profit will ultimately maximise their profit. The main goal of the paper is to answer this question.

The main results of this paper (theorems 1 and 2) include: in Schumpeterian evolution, producers are not only better off (as definitions established by A. Malawski and B. Ciałowicz emphasised), but also the final best production plan may be achieved by realising, step-by-step, the best production plans in the particular stages of evolution.

The results are based on previously obtained theorems for the linear programming problem we described in terms of the Kuratowski convergence (see Denkowska, Denkowski & Kornafel 2017). The next two parts briefly present the definitions and summarise the main properties of this kind of convergence of sets and comment on the requisite mathematical theorems.

The work done for this paper has only a theoretical character and is intended to complete the mathematical description of economic theory of the Schumpeterian evolution in the Debreu model. However, practitioners may also profit from our results: Mathematically, the problem of maximising the producers' profits over

sets, which are polytopes, is a linear programming problem. A well-known numerical method of solving it is called the simplex method, which has a rich bibliography (see e.g. Bertsimas, Tsitsiklis & Tsitsiklis 1997, Bartels & Golub 1969, Karmarkar 1984). The practical conclusion of our result is the following: a convergent sequence of optimal production plans (found in each stage of evolution by the simplex method) has a limit that is the optimum for limiting the problem. Secondly, despite the possible numerical errors in computations, the calculated result is *close* to actual optimum.

## 2. Production System in the Debreu Model

In his monumental 1959 work, G. Debreu described the general equilibrium model using a strictly mathematical apparatus. The linear space  $\mathbb{R}^l$  (with fixed  $l \in \mathbb{N}$ ) is interpreted as the  $l$ -dimensional space of commodities and prices. The model formally consists of production and consumption systems, determined by the behaviour of two groups of agents: producers  $j \in B$  and consumers  $i \in C$  (with a finite number of members in each group). The consumer is characterised by his preferences and budget, his goal being to maximise utility from consumption over the budget set. The  $j$ th producer is described by his production set  $Y_j \subset \mathbb{R}^l$  (determining the production abilities and available technology) and aims to maximise his profit, i.e.:

$$\pi_j^* := \max \pi_j = \max_{y_j \in Y_j} p \cdot y_j.$$

Mathematically, it is a problem of maximising the linear functional  $p \cdot y_j$  over the given set  $Y_j$ . Moreover, when the set  $Y_j$  is a cone (as we see in a moment in the assumptions), the maximisation problem is merely simple linear programming problem.

G. Debreu joins those two sectors and provides the conditions under which it is possible to achieve equilibrium. The assumptions for the production system, which is the heart of the matter here, are reviewed below.

For any producer  $j$ , production set  $Y_j$  satisfies the following conditions (see Debreu 1959):

a)  $Y_j \subset \mathbb{R}^l$  is closed (meaning that if for any the production plan  $y_j^n \in Y_j$  is possible for  $j$ th producer and  $\lim_{n \rightarrow \infty} y_j^n = y_j^0$ , then  $y_j^0 \in Y_j$ , i.e. the limiting plan can also be realised),

b)  $0 \in Y_j$  (the possibility of not producing),

c)  $Y_j \cap \mathbb{R}_+^l \subset \{0\}$  (i.e. free production is impossible without inputs),

d)  $Y_j \cap (-Y_j) \subset \{0\}$  (irreversibility),

- e)  $Y_j + Y_j \subset Y_j$  (any two production plans together are also possible for production),
- f)  $Y_j$  is convex (any combination of two production plans is also possible for production),
- g)  $Y_j$  is a cone with its vertex at (under the assumption of constant returns to scale),
- h)  $\mathbb{R}_+^l \subset Y$ , where  $Y = Y_1 + \dots + Y_n$  (it is possible for all producers together to dispose of all commodities).

As we said, for a given price system  $p \in \mathbb{R}^l$ , a producer aims to maximise profit. It is known that maximisation may not be possible for any price system, so the following is defined:

$$T_j := \{p \in \mathbb{R}^l : \text{there exists } \max_{y_j \in Y_j} (p \cdot y_j)\}.$$

For a production set that is a cone, the set  $T_j$  is its normal cone. The correspondence attaining the maximisers to the given price  $p \in T_j$  is:

$$\eta_j : \mathbb{R}^l \ni p \mapsto \{y_j : p \cdot y_j = \max_{y_k \in Y_k} (p \cdot y_k)\} \subset Y_j.$$

In other words, for  $p \in T_j$   $\eta_j(p) = \arg \max \pi_j(p)$ . In general, if  $Y_j$  is a polytope, the set  $\eta_j(p)$  can be identified with the set of vertices of the production set.

In such a setting, if only  $\bigcap_{j \in B} T_j \neq \emptyset$ , it is possible to prove the existence of the price system, which allows all the producers in the economy to maximise their profits and – in the further perspective of the model and with additional requirements toward the consumption system – the existence of equilibrium in the whole economy. The assumptions listed above remain in force in the next parts of the paper.

### 3. Schumpeterian Evolution

In his 1912 work, J. Schumpeter distinguished two forms of economic life: circular flow and economic development. Circular flow is the state of the economy, in which all the processes go along known trajectories, as determined by economic laws. It could be understood as stagnation. However, an economy undergoes constant evolution, which to J. Schumpeter meant evolution is driven by creative destruction, or the natural process of introducing innovations and eliminating existing goods, production technologies, markets, etc. In 1950, J. Schumpeter wrote: “The fundamental impulse that acts and keeps the capitalistic engine in motion comes from the new consumer goods, the new method of production, the new forms of industrial organization that capitalist enterprise creates.

(...) The opening up of new markets, foreign or domestic, and the organizational development (...) illustrate the same process of industrial mutation – if I may use that biological term – that revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one” (Schumpeter 1950, p. 83).

J. Schumpeter never explained how creative destruction changes circular flow. With his research group at Cracow University of Economics and colleagues from other countries A. Malawski, in a number of publications, has attempted to describe formally how it would happen (see *Innovative Economy...* 2013) and references therein). In particular, B. Ciałowicz and A. Malawski (2011) introduced the definitions of cumulative and innovative extensions of a production system. In the definitions cited below, the “tilde” symbol denotes the quantities, functions and correspondence after the change introduced.

*Definition 1* (Ciałowicz & Malawski 2011). A production system  $\tilde{P} = (\tilde{B}, \mathbb{R}^{\tilde{l}}, \tilde{y}, \tilde{p}, \tilde{\eta}, \tilde{\pi})$  is called a cumulative extension of a production system  $P = (B, \mathbb{R}^l, y, p, \eta, \pi)$ , briefly  $P \subset_c \tilde{P}$ , if:

- 1)  $l \leq \tilde{l}$ ;
- 2)  $p \leq \text{proj}_{\mathbb{R}^l}(\tilde{p})$ , where *proj* denotes orthogonal projection and the inequality between vectors is understood as:

$$p \leq q \Leftrightarrow \forall i: p_i \leq q_i;$$

- 3)  $B \subset \tilde{B}$  and for every  $b \in B$ :
  - a)  $Y_j \subset \text{proj}_{\mathbb{R}^l}(\tilde{Y}_j)$ ,
  - b)  $\eta_j \subset \text{proj}_{\mathbb{R}^l}(\tilde{\eta}_j(\tilde{p}))$ ,
  - c)  $\pi_j(p) \leq \tilde{\pi}_j(\tilde{p})$ .

This definition describes the situation when creative destruction is obtained via the creation of a new good (condition 1, which describes possible extension of the dimension of the commodity and price spaces) or via the introduction of a new technology (condition 3a) and the presence of a new market agent (if  $B \neq \tilde{B}$ ), who actually might be the source of the two listed previously. In particular, if  $l = \tilde{l}$  and  $B = \tilde{B}$ , the projections are identity mappings, so a change may be obtained only by extending the production set  $\tilde{Y}_j$ , but all the previously used technologies are still in use. This extension may be done, for instance, by acquiring additional machines. The whole economic environment therefore remains unchanged and the cumulative extension is intended to model the circular flow in economics. B. Ciałowicz and A. Malawski (2011) also defined a strong cumulative extension of the production system, considering the extensions with respect to different aspects (like number of commodities, price system, etc.) and emphasising the character of circular flow in them.

*Definition 2* (Ciałowicz & Malawski 2011). A production system  $\tilde{P} = (\tilde{B}, \mathbb{R}^l, \tilde{y}, \tilde{p}, \tilde{\eta}, \tilde{\pi})$  is called an innovative extension of a production system  $P = (B, \mathbb{R}^l, y, p, \eta, \pi)$ , briefly  $P \subset_i \tilde{P}$ , if:

- 1)  $l \leq \tilde{l}$ ;
- 2)  $p = \text{proj}_{\mathbb{R}^l}(\tilde{p})$ ;
- 3)  $\exists \bar{j} \in \tilde{B} \quad \forall j \in B$ :
  - a)  $\text{proj}_{\mathbb{R}^l}(\tilde{Y}_{\bar{j}}) \not\subset Y_j$ ,
  - b)  $\text{proj}_{\mathbb{R}^l}(\tilde{\eta}_{\bar{j}})(\tilde{p}) \not\subset \eta_j(p)$ ,
  - c)  $\pi_j(p) \leq \tilde{\pi}_{\bar{j}}(\tilde{p})$ .

*Remark.* The producers who satisfy condition 3 are called innovators.

The definition of innovative extension emphasises the presence of an innovator  $\bar{j}$ , who may introduce a new good or open a new market (if  $l < \tilde{l}$  in condition 1), or may introduce a new technology (method) of production (condition 3a). In contrast to the cumulative extension, the innovative extension exacts the introduction of a new good or new technology and it is possible to rule out an unnecessary or no longer productive technology from the new set of production  $\tilde{Y}_{\bar{j}}$ .

A. Lipieta and A. Malawski (2016) proposed to design economic mechanisms to describe an economic system's evolution from an arbitrary one to its cumulative extension (price-preserving mechanism) and evolution to the innovative extension (qualitative mechanism). They study the relationship between these extensions and use the language of mechanism design developed by L. Hurwicz and S. Reiter (2006). One of the elements of their description is the transition mapping  $T: [0, 1] \rightarrow \wp_j$ , where  $T(0) = P_j$  and  $T(1) = \tilde{P}_j$ . The set  $\wp_j$  denotes all the possible production systems. The authors showed (Lipieta & Malawski 2016) that it is possible to design transitions that will lead to cumulative and innovative extensions, respectively.

A question that does not appear in the analysis is whether or not a producer who in each stage of those transitions maximises his profit will ultimately maximise profit. From a mathematical point of view, the answer is not obvious. Definitions 1 and 2 assure only that the final profit will not be smaller than it was at the beginning. In the next part of the paper, a theorem showing that this convergence is assured is formulated using the language of Kuratowski convergence, which makes it possible to describe interesting phenomenon relatively easily. To clarify the issue, the necessary mathematical definitions and theorems are written down in the 5th and 6th sections.

#### 4. The Dynamics of the Producers' Optima in Schumpeterian Evolution

Let us consider the mapping (transition)  $T: [0,1] \rightarrow \mathcal{P}_j$ , which describes the evolution of the production system  $P_j = T(0)$  to another state  $\tilde{P}_j = T(1)$ . In particular, the final state may be a cumulative or innovative extension of the production system  $P_j$ .

For any time moment  $t \in [0,1]$  we deal with the production system  $P_j^t = (B, \mathbb{R}^l, y^t, p^t, \eta^t, \pi^t)$ , in which the  $j$ th producer maximises his profit  $\pi_j^t$  over the production set  $Y_j^t$ . The price vectors  $(p^t)_t$  by assumption create a convergent sequence and  $\lim_{t \rightarrow 1} p^t =: p^1$ . The prices  $p^t$  and  $p^1$  are assumed to be such vectors that  $\eta_j^t(p^t) \neq \emptyset$  and  $\eta_j^1(p^1) \neq \emptyset$ . Then, as shown by arguments from the Debreu model, there exists  $y_j^{t*} \in \eta_j^t(p^t)$ .

In the theorems cited in part 6, all the sets are subsets of the same vector space  $\mathbb{R}^l$ . Here, due to the important role of dimension (determining the number of goods in the market), constant  $l$  is quite an unpleasant inconvenience and would limit us only to the case when the dimension is not increased in the Schumpeterian evolution. We overcome this difficulty by the following argument. When  $(l^t)$  is a non-constant sequence, we assume it to be bounded (which is economically reasonable as it is impossible to create or develop an infinite number of goods or technologies in a given time period). It is then enough to define  $l := \sup_{t \in [0,1]} l^t$  and identify the vectors and sets from  $\mathbb{R}^{l^t}$  with their embeddings in  $\mathbb{R}^l$ . To be clear:

- 1) the vector  $p^t \in \mathbb{R}^{l^t}$  will be identified with the vector

$$\tilde{p} := p^t \times \underbrace{\{0\} \times \dots \times \{0\}}_{l - l^t \text{ times}} \in \mathbb{R}^l,$$

- 2) the production set  $Y_j^t \subset \mathbb{R}^{l^t}$  will be identified with the set

$$\tilde{Y}_j^t := Y_j^t \times \underbrace{\{0\} \times \dots \times \{0\}}_{l - l^t \text{ times}} \in \mathbb{R}^l,$$

- 3) the correspondence  $\eta_j^t(p^t) \subset \mathbb{R}^{l^t}$  will be identified with the set

$$\tilde{\eta}_j^t(\tilde{p}) := \eta_j^t(p^t) \times \underbrace{\{0\} \times \dots \times \{0\}}_{l - l^t \text{ times}} \in \mathbb{R}^l.$$

In such a setting, it is immediately clear from the definition of the canonical scalar product of vectors that the value of the profit function does not depend on whether we consider the original elements  $p^t$  and  $y_j^{t*}$  or the corresponding elements from the embeddings.

By assumption (g) in the Debreu model with the constant returns to scale (see part 2), production sets  $Y_j^t$  are cones, and therefore special cases of polytopes with one vertex and two faces.

*Theorem 1.* Assume that  $\lim_{t \rightarrow 1} Y_j^t = Y_j^1$  in the Kuratowski sense. Then:

- 1)  $Y_j^1$  is a cone,
- 2) there exists a subsequence of the sequence of vertices of sets  $Y_j^t$ , i.e. the sequence of vectors  $(v_j^t)$ , which is convergent to the vertex  $v_j^1$  of set  $Y_j^1$ .

*Proof.* As mentioned above, cones are special cases of polytopes with one vertex. There is therefore, obviously,  $\#Y_j^t = 1$ , so the second condition from the theorem 3 is satisfied. This theorem immediately provides the second part the statement 2 in theorem 1. Because  $\#Y_j^t = 1$  and the set is a polytope, it is clear that it is also a cone, which completes the proof of the statement 1 in theorem 1.  $\square$

The first theorem shows that the Kuratowski limit of the sequence of production sets is a good candidate for the production system in the final state. The optimal plans are then indeed convergent to the optimal plan in the final system. This theorem characterises very well the convergence for both cumulative and innovative extensions of the production system.

*Theorem 2.* The sequence of sets  $(\eta_j^t(p^t))_t$  is convergent to  $\eta_j^1(p^1)$  in the Kuratowski sense.

*Proof.* The theorem is the consequence of theorem 4. If necessary, the economic system is considered in the space  $\mathbb{R}^l$  with  $l := \sup_{t \in [0,1]} l^t$ , as described above.  $\square$

In the second theorem, the convergence of maximisers is shown also in the case of price evolution. Recall that this was present in the considerations on circular flow and cumulative extensions of the production systems.

## 5. Mathematical Appendix: Kuratowski Convergence

In this part of the paper we are going to recall briefly the definition of the Kuratowski limit of a sequence of sets. Due to the fact that our economic model is set in  $n$ -dimensional real space  $\mathbb{R}^n$ , the definition and basic facts are formulated in the simplified version of this vector space. Those results are known and the reader may find more details in (Dal Maso 1993).

The Kuratowski convergence is a generalisation of the convergence in the Hausdorff metric to the convergence of closed sets.

Hausdorff sought to extend his metric (defined for nonempty compact subsets of a metric space) to closed sets. In the beginning of the 20th century, P. Painlevé

introduced the concept of upper and lower limits of a sequence of closed subsets in a metric space. The resulting convergence was later studied by several mathematicians (L. Zoratti, C. Zarankiewicz) but it was K. Kuratowski who first prepared a thorough exposition of this theory in his monumental book *Topologie* (Kuratowski 1961). It soon became apparent that this natural convergence of closed sets is a most useful tool for optimisation. In particular, the famous De Giorgi's  $\Gamma$ -convergence of extended-valued functionals on a topological space is precisely the Kuratowski convergence of their epigraphs. It is a powerful variational convergence in that both the minima and minimisers converge to the minimum (respectively, the minimiser) of the limiting functional.

Recall the basic definitions and facts. Let  $E \subset \mathbb{R}^k \times \mathbb{R}^n$  be a nonempty set and  $\pi: \mathbb{R}^k \times \mathbb{R}^n \ni (t, x) \mapsto t \in \mathbb{R}^k$  the natural projection. Fix an accumulation point  $t_0 \in \pi(E) \setminus \{t_0\}$ . We write  $E_t = \{x \in \mathbb{R}^n: (t, x) \in E\}$  for the section of  $E$  at  $t$ .

*Definition 3.* We define the lower and upper Kuratowski limits of the family (net)  $(E_t)$  when  $t \rightarrow t_0$  when respectively as the sets:

$$x \in \liminf_{t \rightarrow t_0} E_t \Leftrightarrow \text{for any neighbourhood } U \text{ of } x, \text{ there is a neighbourhood } V \text{ of } t_0 \\ \text{such that } U \cap E_t \neq \emptyset, \quad \text{for all } t \in V \cap \pi(E) \setminus \{t_0\};$$

$$x \in \limsup_{t \rightarrow t_0} E_t \Leftrightarrow \text{for any neighbourhood } U \text{ of } x \text{ and any neighbourhood } V \text{ of } t_0 \\ \text{there is a point } t \in V \cap \pi(E) \setminus \{t_0\} \quad \text{such that } U \cap E_t \neq \emptyset.$$

Clearly,  $\liminf_{t \rightarrow t_0} E_t \subset \limsup_{t \rightarrow t_0} E_t$ . If the converse inclusion also holds, we denote the resulting set by  $\lim_{t \rightarrow t_0} E_t$  and call it the Kuratowski limit of  $E_t$  when  $t \rightarrow t_0$ . Therefore,  $(E_t)$  converges to some set  $F$  as  $t \rightarrow t_0$ , iff

$$\limsup_{t \rightarrow t_0} E_t \subset F \subset \liminf_{t \rightarrow t_0} E_t.$$

We also then write that  $E_t \xrightarrow{K} F$  (when  $t \rightarrow t_0$ ).

It is easy to see that the upper and lower limits are closed sets that remain unchanged, if we replace the sets  $E_t$  by their closures.

Observe that a sequence of sets  $E_v \subset \mathbb{R}^n$  may be treated as the  $t$ -sections of the set  $E := \bigcup_{v=1}^{+\infty} \left\{ \frac{1}{v} \right\} \times E_v \subset \mathbb{R} \times \mathbb{R}^n$ . Then the convergence of the sequence  $(E_v)$  when  $v \rightarrow +\infty$  is simply the convergence of the sections  $(E_t)$  when  $t \rightarrow 0$ . Clearly,  $\liminf_{v \rightarrow +\infty} E_v$  consists in this case of all the possible limits of convergent sequences chosen point by point:  $x_v \in E_v$ , while  $\limsup_{v \rightarrow +\infty} E_v$  is the set of all the limits of convergent subsequences  $x_{v_k} \in E_{v_k}$ .



The most important feature of this convergence is that in the setting introduced above it is metrisable and compact (C. Zarankiewicz).

## 6. Mathematical Appendix: Convergence of Optima in a Linear Programming Problem

In linear programming, the vertices of a given linear polytope play an important role. As a matter of fact, if a linear functional attains its extremum on a linear polytope, then it attains it on the boundary and in particular in a vertex (provided, of course, that the set of extremal points  $E^*$ , i.e. vertices, is nonempty). Geometrically, this is captured by the position of the gradient of the functional with respect to the normal cones computed at the vertices.

One natural question is what happens when we allow some evolution in time (either of the constraints, i.e. of the polytope, or of the functional). Theorem 3 below gives a sufficient condition for the limit set of a Kuratowski-convergent sequence of linear polytopes to be a linear polytope. Moreover, we prove that the normal cones at the vertices converge to normal cones. In the non-compact case, the assumption of a uniform bound on the number of extremal points is not sufficient to obtain a polytope as a limit (see our paper Denkowska, Denkowski & Kornafel 2017).

*Theorem 3.* Assume that  $\lim_{v \rightarrow +\infty} E_v = E$ , where  $E_v \subset \mathbb{R}^n$  are linear polytopes. Let  $f_k(E_v)$  denote the number of  $k$ -dimensional faces of the polytope  $E_v$  (here  $k \in \{0, \dots, \dim E_v\}$ , where the dimension is the dimension of the affine envelope).

If there is a constant  $M > 0$  such that either  $f_{\dim E_v - 1}(E_v) \leq M$  for all  $v$ , or  $E$  is compact and  $\#E_v^* \leq M$ , then:

- 1)  $E$  is a linear polytope, too,
- 2)  $f_{\dim E - 1}(E) \leq M$  or  $\#E^* \leq M$ , respectively,
- 3) for any  $a \in E^*$  there is a sequence  $a_v \in E_v^*$  converging to such that the normal cones  $N_{a_v}(E_v)$  converge to  $N_a(E)$  in the sense of Kuratowski.

Thanks to the geometric characterisation of minimisers using normal cones, we obtain Theorem 4 concerning the convergence of minimisers.

*Theorem 4.* Assume that the vectors  $c_v \in \mathbb{R}^n$  converge to  $c$  and let  $M_v$  denote the set of minimisers of  $f_v(x) = c_v^T x$  in  $E = E_{A,b}$ . The sequence  $(M_v)$  converges then in the sense of Kuratowski to the set  $M \subset E$  being the set of minimisers for the limiting functional  $f(x) = c^T x$ .

## 7. Conclusions

This paper presented the theorems which complete the formal description of Schumpeterian evolution of the Debreu model. We showed that if only the price system evolves to some final state and the innovations (determining the production sets  $Y_j$ ) are such that the sequence  $(Y_j)$  is convergent in the Kuratowski sense to some set  $\tilde{Y}$ , then the producers in the final production system are not only better off, but they achieve the maximal possible profit. It is also possible to approach the plan providing this maximal profit by a sequence of optimal production plans in the process of evolution. The character of Kuratowski convergence has additional practical implications: a numerical scheme (simplex method) applied to this problem will provide the result as optimum or close to optimum (due to possible numerical errors).

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## **Optima producentów w ewolucji Schumpetera**

(Streszczenie)

W artykule rozważany jest system produkcji w modelu równowagi ogólnej Debreu. Zgodnie z teorią J. Schumpetera w celu wybicia gospodarki ze stagnacji konieczne jest wprowadzenie innowacji. W ich wyniku otrzymuje się ciągi planów optymalnych, odpowiadających każdemu etapowi wprowadzania innowacji. W pracy scharakteryzowano proces zmiany optymalnych planów produkcji i opisano warunki, przy których zagwarantowana jest ich zbieżność w taki sposób, że stan graniczny realizuje optimum stanu finalnego.

**Słowa kluczowe:** model Debreu, system produkcji, ewolucja Schumpetera, innowacje, zbieżność Kuratowskiego.

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# Market Commentaries and Stock Prices in Poland: A Text Mining Approach\*

## Abstract

From a theoretical point of view, the scope and quality of available information determines the market efficiency and, thus, investors' decisions. However, an excessive amount of information leads to information overload. In the case of textual data, advanced analytical methods must be applied to identify some regularities and trends within the analysed text corpora. Text mining may be useful in supporting the decision-making process.

The paper examines the interdependencies between market commentaries and stock prices. More specifically, it verifies the linguistic characteristics of opinions distributed by institutional investors (investment fund company) and their intertemporal links to the price movements on the Warsaw Stock Exchange.

The results indicate that: 1) there is no significant linguistic difference between market commentaries written after weeks of relatively low and relatively high rates of returns on the Warsaw Stock Exchange; 2) the linguistic content of selected market commentaries does not have a predictive value for the Polish stock market; 3) commentaries with

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\* This research has been financed with funds granted to the Faculty of Finance and Law of the Cracow University of Economics from a subsidy for maintaining research potential.

a one-week time difference linguistically differ less than the commentaries with two or more weeks' time difference.

**Keywords:** information, stock market prediction, text mining, analysts recommendation, market commentaries.

**JEL Classification:** G14, G17.

## 1. Introduction

The growing quantity of information produced, the speed of its distribution and the ease of its accessibility in recent years, combined with ongoing progress in information and communication technology (ICT) are some of the forces shaping today's financial markets – and posing a real challenge for all market participants making investment decisions in a globalised and interconnected environment. From a theoretical point of view, the scope and quality of available information determines the market efficiency and, thus, investors' decisions. However, an excessive amount of information leads to information overload, which makes searching for relevant sets of information a complex and time-consuming process requiring one to simultaneously explore and process various textual and non-textual data sets. Text information ranges from short text tweet messages and market commentaries to multi-page analyses and recommendations disseminated by both, private and public entities.

The volume of information to be analysed by investors justifies the use of advanced analytical methods to identify regularities and trends within the text corpora. Text mining, a programmed multi-stage and multidirectional procedure, may be useful in supporting decision-making by providing high-quality information obtained with relatively little human effort. Regardless of the approach (e.g. semantic parsing, bag of words), it enables the automatic sorting of collected text data and extraction of its essential features, which are then subject to detailed analysis. The text reorganisation and transition from an unorganised to an organised state (through stemming, white space elimination, lower case conversion, synonym replacement, tagging, etc.) may therefore provide investors with a potentially useful information.

The paper examines the interdependencies between market commentaries and stock prices. More specifically, we verify the linguistic characteristics of selected text corpora distributed by institutional investor (investment fund company) and their intertemporal links to the price movements at the Warsaw Stock Exchange. Our main focus lies in diagnosing the linguistic differentiation of market commentaries under different market conditions and checking whether a lexical (language style-related) analysis of analyst opinions may create an informational

advantage in the stock market. Based on observation of the market, we postulate following hypotheses:

1. Market commentaries are written in a different way after weeks of relatively low or relatively high observed rates of return.
2. The content of market commentaries has a predictive value.
3. Commentaries with shorter time difference linguistically differ less than the commentaries with a longer time difference.

## 2. Literature Review

The question of how public or private information affects asset price behaviour has for decades been a mainstream issue in financial market research. The problem is multifaceted and different perspectives (legal, behavioural, microstructural, technological, etc.) have been taken to examine their mutual relations and dependencies, both theoretically and empirically.

The main emphasis lies in demonstrating the impact of endogenous or exogenous information on market efficiency (Fama 1970, Boya 2013) and the consequences of the informational asymmetry between market participants (Kyle 1985, Wang & Wang 2017). The market responses to new information are usually investigated in the context of stock splits (Arbel & Swanson 1993, Kalay & Kronlund 2013), dividends (Kaestner & Liu 1998, Dasilas & Leventis 2011) or earning announcements (Basu *et al.* 2013). Due to the advances in ICT, written communication is increasingly distributed online through information services, official websites (e.g. of stock exchanges) or social media platforms (Azar & Lo 2016). This makes it easily accessible and ready for further processing. However, high quality analytical data (e.g. real-time quotes and news) is costly; it is particularly the costs associated with acquiring information that pose a significant barrier for small investors. This hampers overall market efficiency and prompts the need for appropriate regulations, which should primarily be introduced in relation to data being offered for sale by stock exchanges (Easley, O'Hara & Yang 2016).

One of the popular sources of exogenous information for investors is the opinions and recommendations of analysts, which can induce noticeable stock price movements (Jagadeh & Kim 2006, Ramnath, Rock & Shane 2008, Mielcarz 2015). The scope of price reactions depends on the recommended trading direction and tends to lead to greater price drifts in case of "sell" than "buy" intimations (Womack 1996). In general, the informativeness of analysts' recommendations translates into abnormal returns mostly in the short term than in the long term (Wnuczak 2015, Murg 2016).

From a methodological point of view, event studies or analyses of informational flux play a predominant role in testing the information-price relationship in the case of exogenous information, and various predictability tests based on random walk, long memory, seasonal effects and trading volume for endogenous information (Boya 2013). Due to the problem of interdisciplinarity and the enormous production and output of news in recent years, approaches utilising linguistics, machine learning and behavioural economics-related methods have also found use in searching for interdependencies between textual information and price changes (Khadjeh Nassirtoussi *et al.* 2014). In this context, text mining tools have become increasingly popular, especially as they prove their usefulness in verifying market sentiment (Ishijima, Kazumi & Maeda 2015) and predicting stock returns (Tetlock, Saar-Tsechansky & Macskassy 2008, Sun, Lachanski & Fabozzi 2016), corporate earnings (Tetlock, Saar-Tsechansky & Macskassy 2008) and market volatility (Mamaysky & Glasserman 2016). The analytical potential of these tools is expected to gain in importance in future financial market research.

### 3. Data

108 weekly market commentaries disseminated by the analyst team of a Polish investment fund company have been examined. Each commentary is freely available at the company's web site and additionally distributed through information services and financial portals. It is written in English, saved as a PDF file, covers one page and is organised in two subsections: market summary and technical analysis. Our analysis covers the period 9.06.2014 to 13.03.2017. Additionally we used the WIG time series ([www.stooq.pl](http://www.stooq.pl)) covering the same period.

### 4. Methodology and Results

We applied standard text mining tools to verify the hypotheses. First each commentary was read into analysis environment, a *tm* package of R (Feinerer, Hornik & Meyer 2008, Feinerer & Hornik 2017). Each verb was then tagged using the *wordnet* dictionary and relevant R interface (Fellbaum 1998, Feinerer & Hornik 2017). Tagging provides the part-of-speech (POS) information. For example, \JJR denotes adjectives in their superlative form, while \NNPS is for plural proper nouns. For a systematic description of all tags used, see (Marcus, Marcinkiewicz & Santorini 1993). We constructed the following metrics for further analysis: the percentage share of adjectives in the whole text,

the percentage share of adjectives in comparative and superlative forms to all adjectives, the percentage share of adverbs in the whole text, percentage shares of adverbs in comparative and superlative form in all adjectives, and, respectively, the percentage share of verbs, nouns, foreign words and numbers in the whole text. We compared these metrics for two separate groups of weekly commentaries. The first group encompasses those commentaries written after a relatively poor WIG performance (1/3 of all commentaries with the lowest values of WIG weekly return rates), while the second group encompasses those written after a relatively strong WIG performance (1/3 of all commentaries with the highest values of WIG weekly return rates). The results are presented in Table 1. Commentaries written after the low-rate-of-return weeks tend to have more adverbs (at a significant level) and fewer numbers (non-significant) than those written after weeks with a high rate of return.

Table 1. *T* Test Comparison Results: Statistical Value and Relevant *p* Value between Commentaries Written after a Relatively Low WIG Rate of Return (*x*) and after a Relatively High WIG Rate of Return (*y*) Based on the Part-of-speech Derived Metrics

Metric	Statistic	<i>p</i> Value	<i>x</i> Mean	<i>y</i> Mean
WIG	-14.8939	0.0000	-1.59%	1.89%
Words number	0.625701	0.5335	701.36	693.97
pct_adjective	-0.06441	0.9488	9.40%	9.42%
pct_adjective_comp	0.965701	0.3377	9.38%	8.44%
pct_adverb	2.1015	0.0392	5.30%	4.84%
pct_adverb_comp	0.310867	0.7568	3.85%	3.60%
pct_noun	-0.03454	0.9725	32.52%	32.53%
pct_verb	0.707422	0.4817	12.98%	12.79%
pct_foreign	-0.58293	0.5621	0.02%	0.03%
pct_numbers	-1.21507	0.2285	3.68%	3.87%

Source: the authors' own calculations.

We verified this result based on other commentaries' classification in other groups. Namely, we used the Dow Jones Industrial Average (DJIA) index for classification into weeks with relatively low or high rates of return. The results are presented in Table 2. The differences are not significant (less significant than in the case of WIG-based commentary classification) which is to be expected as the commentaries contain a summary of the financial markets from the previous weeks on the world market but concentrate on the situation on the Warsaw Stock Exchange.



Table 2. *T* Test Comparison Results: Statistical Value and Relevant *p* Value between Commentaries Written after a Relatively Low DJIA Rate of Return (*x*) and after a Relatively High DJIA Rate of Return (*y*) based on the Part-of-speech Derived Metrics

Metric	Statistic	<i>p</i> Value	<i>x</i> Mean	<i>y</i> Mean
DJIA	-4.71202	0.0000	-0.44%	1.09%
Words number	0.668226	0.5062	701.70	694.34
pct_adjective	0.039847	0.9683	9.44%	9.43%
pct_adjective_comp	1.36671	0.1756	9.60%	8.35%
pct_adverb	1.647495	0.1040	5.20%	4.86%
pct_adverb_comp	0.737309	0.4631	4.32%	3.71%
pct_noun	0.109566	0.9131	32.59%	32.55%
pct_verb	0.460124	0.6469	12.93%	12.82%
pct_foreign	-0.23579	0.8144	0.02%	0.02%
pct_numbers	-0.83012	0.4090	3.73%	3.85%

Source: the authors' own calculations.

Table 3. *T* Test Comparison Results: Statistical Value and Relevant *p* Value between Commentaries Written before a Relatively Low WIG Rate of Return (*x*) and before a Relatively High WIG Rate of Return (*y*) based on the Part of Speech-derived Metrics

Variable	Statistic	<i>p</i> Value	<i>x</i> Mean	<i>y</i> Mean
wig_f	-17.4536	0.0000	-1.57%	1.99%
Words number	1.014274	0.3140	701.5556	691.5278
pct_adjective	0.440511	0.6611	9.47%	9.35%
pct_adjective_comp	0.002455	0.9980	9.92%	9.92%
pct_adverb	0.220446	0.8262	5.00%	4.95%
pct_adverb_comp	1.037893	0.3029	4.46%	3.60%
pct_noun	0.125138	0.9008	32.58%	32.54%
pct_verb	-0.51402	0.6090	12.83%	12.93%
pct_foreign	-0.45144	0.6531	0.02%	0.03%
pct_numbers	-0.82598	0.4116	3.69%	3.84%

Source: the authors' own calculations.

The last test verified if the linguistic content of the commentary may provide and disclose some information on the next week's rate of return. For this purpose, the classification of commentaries was put into a group that encompasses commentaries written before weeks of relatively low WIG rates of return and others written before weeks with relatively high WIG return rates. The results

are presented in Table 3. No significant differences between these two groups of commentaries were observed. The results are presented in Table 3.

We also compare how similar the commentaries are. For this purpose we first remove punctuation, stop words and numbers, change all the words to lower case and conduct a stemming procedure. We then construct the term document matrix, which contains for each word stem and each respective document the number of occurrences. We limit the number of word stems to those that had occurred at least 3 times on the global level. Then the differences in terms of the cosine and Jaccard distances are calculated, as recommended in the literature (Strehl, Ghosh & Mooney 2000). Then we compare the average (cosine and Jaccard) distance within the class of commentaries that differ by 1 week with the average distance within the class of commentaries that differ by 2 weeks and so forth. The results are presented in Table 4. We can observe that commentaries that are 1 week different are more similar to each other than the commentaries that are 2 or more weeks different.

Table 4. *T* Test Comparison Results: Statistical Value and Relevant *p* Value between the Cosine Distance of Commentaries that Differ by *i* Weeks and the Cosine Distance of Commentaries that Differ by *i* + 1 Weeks (the First Column Provides the Information on Particular *i* Value and Is in Form of: *i* / *i* + 1)

Weeks	Cosine Distance	<i>t</i> Test	<i>p</i> Value
1 / 2	0.318755	-2.21405	0.0285
2 / 3	0.34799	-0.06051	0.9518
3 / 4	0.348819	-0.11756	0.9066
4 / 5	0.350426	0	1.0000
5 / 6	0.350426	-0.08028	0.9361
6 / 7	0.351529	0	1.0000
7 / 8	0.351529	-0.0854	0.9321
8 / 9	0.352711	0	1.0000
9 / 10	0.352711	0	1.0000
10 / 11	0.352711	0	1.0000
11 / 12	0.352711	0	1.0000
12 / 13	0.352711	-0.03817	0.9696

Source: the authors' own calculations.

We also validated the result based on Jaccard distance metrics. The results presented in Table 5 corroborate the difference in the similarities between commentaries that differ by 1 week and those that differ by 2 or more weeks at a significance level of 0,1335.

Table 5. *T* Test Comparison Results: Statistical Value and Relevant *p* Value between Jaccard Distance of Commentaries that Differ by *I* Weeks and the Jaccard Distance of Commentaries that Differ by *i* + 1 Weeks (the First Column Provides the Information on Particular *I* Value and Is in Form of: *i* / *i* + 1)

Weeks	Jaccard Distance	<i>t</i> Test	<i>p</i> Value
1 / 2	0.639781	-1.50915	0.1335
2 / 3	0.659714	-0.13763	0.8907
3 / 4	0.66155	-0.07453	0.9407
4 / 5	0.66255	0	1.0000
5 / 6	0.66255	-0.08982	0.9286
6 / 7	0.663765	0	1.0000
7 / 8	0.663765	-0.08178	0.9349
8 / 9	0.664879	0	1.0000
9 / 10	0.664879	0	1.0000
10 / 11	0.664879	0	1.0000
11 / 12	0.664879	0	1.0000
12 / 13	0.664879	-0.03486	0.9722

Source: the authors' own calculations.

Table 6. A Comparison of the *T* Test Results: The Statistical Value and Relevant *p* Value between the Cosine Distance of Commentaries of Classes *i* and *i* + 1 (the First Column Provides the Information on the Particular *I* Value and Takes the Form: *i* / *i* + 1; the Description of Commentaries Classes in a Text)

Group	Mean <i>i</i>	Mean <i>i</i> + 1	<i>t</i> Test	<i>p</i> Value
1 / 2	0.314184	0.309868	0.198287	0.8438
1 / 3	0.314184	0.329076	-0.66034	0.5122
2 / 3	0.309868	0.329076	-0.99655	0.3237

Source: the authors' own calculations.

The third analysis compares the similarity of the commentaries with a one-week difference. Each pair of such commentaries is classified into three separate classes based on the previously observed WIG weekly rate of return for both commentaries of a particular pair. Namely, a given pair is classified as a 1 if one of the commentaries is preceded by a relatively low rate of return (1/3 with lowest return), while the second commentary is preceded by a relatively high rate (1/3 with highest return). A given pair is classified as a 2 if both commentaries are preceded by the same WIG rate of return (relatively low -1/3 with the lowest return, middle -1/3 with a middle return and relatively high -1/3 with the highest

return), respectively. Otherwise a given pair is classified as a 3. Table 6 compares the results, showing that there were no significant differences.

## 5. Conclusions

The growing amount of information distributed to stock market investors requires pertinent analytical and computational tools enabling data processing and supporting the decision-making process. Text mining can be a useful way to search for patterns and trends within textual stock market commentaries and analysts' opinions.

The analysis of the interdependencies between selected documents prepared by one of the institutional investors operating on the Polish stock market and stock price behaviour was carried out using basic text data processing operations (e.g. speech tagging) and similarity measures (cosine and Jaccard distance).

Our results indicate that: 1) there is no significant linguistic difference between market commentaries written after weeks of relatively low and relatively high rates of return on the Warsaw Stock Exchange; 2) the linguistic content of selected market commentaries does not have a predictive value for the Polish stock market; 3) commentaries with a one-week time difference linguistically differ less than the commentaries with two or more weeks' time difference.

Our further research will involve a larger set of textual data from different information sources (e.g. information services, social media) and apply advanced text mining tools to comprehensively verify their potential to provide informative value to investors. Having information gained from the text mining tools may translate into an advantage in the stock market.

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## **Komentarze rynkowe a ceny akcji w Polsce – analiza z wykorzystaniem metod eksploracji danych tekstowych**

(Streszczenie)

Z teoretycznego punktu widzenia zakres i jakość dostępnych informacji determinuje efektywność rynku, a tym samym wpływa na decyzje inwestycyjne inwestorów. Jednakże duża ilość informacji nie przekłada się wprost na poprawę tej efektywności, wymaga natomiast odpowiedniej selekcji, segregacji oraz strukturyzacji w celu ekstrakcji potencjalnych sygnałów kupna lub sprzedaży instrumentów finansowych. W przypadku informacji tekstowych realizowanie tych działań może zostać usprawnione poprzez zastosowanie komputerowych metod eksploracji danych tekstowych (*text mining*).

Przedmiotem analizy jest weryfikacja parametrów lingwistycznych komentarzy rynkowych oraz ocena ich powiązań z wahaniami cen akcji notowanych na Giełdzie Papierów Wartościowych w Warszawie. Badania z wykorzystaniem wybranych metod i mierników analizy danych tekstowych (m.in. znakowanie części mowy, odległość cosinusowa, Jaccarda) prowadzone są na bazie korpusów tekstowych opracowań analitycznych, sporządzanych regularnie w odstępach tygodniowych przez profesjonalny zespół analityków rynkowych jednego z towarzystw funduszy inwestycyjnych dla polskiego rynku akcji.

Wyniki badań wskazują, że: 1) nie występuje statystycznie istotne zróżnicowanie językowe tekstów komentarzy sporządzanych po zakończeniu tygodni o relatywnie niskich oraz tygodni o relatywnie wysokich stopach zwrotu na Warszawskiej Giełdzie Papierów Wartościowych, 2) zawartość językowa wybranych komentarzy rynkowych nie ma wartości predykcyjnej dla polskiego rynku akcji, 3) komentarze z jednotygodniową różnicą czasową różnią się pod względem lingwistycznym mniej niż komentarze z różnicą czasu dwóch lub więcej tygodni.

**Słowa kluczowe:** informacja, przewidywanie cen akcji, analiza danych tekstowych, rekomendacje analityków finansowych, komentarze rynkowe.



| *Kamila Prucia*

# Partial Effectiveness of Scientific Projects Financed through Public Funds\*

## Abstract

One type of effectiveness is partial effectiveness determined while the project is on-going. It takes into account the progress of work being performed and the costs incurred. The article analyses and assesses the effectiveness of tools for monitoring the partial effectiveness of scientific projects. The specific goal is to determine the relationship between the current evaluation and partial effectiveness in relation to scientific projects.

Scientific projects financed within National Centre for Research and Development STRATEGMED programme is the subject of the analysis. An analysis of the funds disbursed to beneficiaries by NCRD has been performed with the use of the dynamic method (streaming), which consists in an analysing the cash flow between two subjects. The monographic method was used to analyse tools for monitoring partial effectiveness, enabling an examination of their features and components in detail.

**Keywords:** project, evaluation, effectiveness, beneficiary.

**JEL Classification:** O220, O320, O200.

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| \* This paper presents the results of a research project financed from Ministry of Science and Higher Education funds granted to the Faculty of Finance and Law at the Cracow University of Economics for research projects implemented by young scientists and doctoral research fellows.



## 1. Introduction

The research done for this article was undertaken for two reasons. The first is that analysing the financing of science as a public task of a strategic nature for the state is important. The second is that there is a need to monitor and define ways of measuring and assessing the effectiveness of public funds intended for financing science. A difficulty in measuring the effectiveness of scientific projects is that this form of research is associated with uncertainty and risk. A significant part of research on effectiveness of the funding is carried out *ex post*. However, it should be considered whether the effectiveness of spending public funds should not be studied earlier – at the project implementation stage. In the last 20–30 years, there have been numerous changes in state management in the global environment. In terms of public expenditure as an instrument, how the economic and financial measures system is approached has changed. Science has developed significantly, as has practical experience “accountable management” in the public sector. It is a management model based on a measurement system which implies the need to implement the efficiency measurement function for the public sector (Lubińska 2007). The assessment of the effectiveness of public task implementation is systematised on two levels: macroeconomic and microeconomic. Public spending should be subject to modern procedures that result from the theory and practice of management, including (Owsiak 2004):

- budgeting by needs to plan expenses as needed,
- measuring the effects of public expenditure, i.e. measuring the achievements of units performing public tasks,
- taking into account achievements in the quality of public services during measurement,
- comprehensively evaluating budget programmes.

In recent decades, the most common method used to measure the effects and benefits of public expenditure has been the Data Envelopment Analysis, or DEA, whose progenitors referred to the productivity. The quotient of a single effect and a single expenditure was adopted as the measure of technical effectiveness (Owsiak 2015). J. Jakuszewicz (2015), examined how management instruments are shaped in the scientific research sector and how productivity of scientific units in Poland are measured in the context of parametric assessment of the Ministry of Science and Higher Education, using the DEA method. The research concludes that including a variable to characterise financial resources yields a more comprehensive explanation of the level of effects achieved. Financial expenditures on scientific activities determine the scientific effects achieved by researchers.

The general goal of the article is to analyse and assess the effectiveness of tools for monitoring the partial effectiveness of scientific projects. The specific goal is

to determine the relationship between current evaluation and partial effectiveness in relation to scientific projects. The hypothesis is that on-going evaluation determines the partial effectiveness of a research project. For the needs of the study, selected evaluation tools used by the National Centre for Research and Development (NCRD) at the stage of carrying out a research project are analysed.

## 2. Using Public Funds to Finance Scientific Projects

There are two forms of funding for science in the public sector – institutional funding and funding projects. The first consists in transferring grants to beneficiaries on the basis of performance indicators or based on negotiations, or direct selection of projects as well as programmes which are to be carried out. The second form, funding projects, consists in transferring grants of a limited size, budget and for a limited time to researchers or research teams, usually on the basis of project proposals describing anticipated research activity in terms of competition. In contrast to institutional funding, funding projects endow organisations with greater control. The current increase in financing projects at the cost of institutional financing is justified by the conviction that produces greater knowledge and better adjusts research to social and economic needs (*Polityka naukowa...* 2015).

The word “project” comes from the Latin word *proiectus*, “before an action”. It is translated as the presentation of a description of a solution for a task to be performed in the future. According to the definition provided by the Project Management Institute, a project is temporary and its aim is to create a unique product (Stabryła 2006).

According to the definition of T. L. Young, a project is a specific set of operations designed to accomplish a singular goal, managed in an organised way. It has a defined beginning and end in time, for obtaining specific results which fulfill the needs of an organisation arising from its on-going business plans and its general concept of development (Drobniak 2005).

In a broad sense, a project is connected with a base solution (innovation), constituting a product characterised by an innovative form and construction, which will enable completion of an anticipated undertaking and the investment, operational and public task (Stabryła 2006). According to another definition, a project is a network of actions with precisely defined beginning and end in time (Spinner 1992).

A project should serve the purpose of e.g. (Peregrinus 1990):

- providing essential infrastructure services and/or the further development of an institution,

- increasing the value of goods by producing or processing them into a more valuable product,
- developing a nation's natural resources.

According to M. D. Rosenau, a project is characterised by four features which distinguish it from other anticipated and managed activities, including three-dimensionally defined objectives, uniqueness, use of resources and implementation within an organization. Three-dimensional objectives refer to the equivalent perception of a project's results, the time schedule and budget. A project's scope covers the anticipated and deliberate use of financial resources as well as those which are combined with human engagement. G. D. Oberlander defined a project as activity undertaken to achieve specific results by its beneficiary (Gorlewski 2015).

A scientific project as an undertaking being the subject of a grant agreement between executive agencies (National Centre for Research and Development, National Science Centre) and a beneficiary is a fundamental tool for financing science.

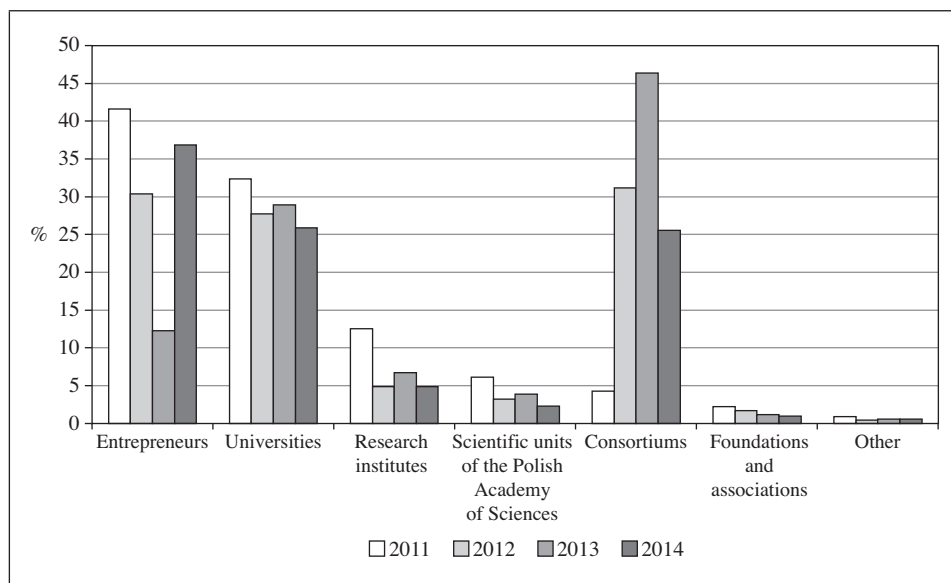


Fig. 1. Share of Scientific Projects Carried out by Individual Beneficiaries to the Total Number of Scientific Projects Implemented in the Period 2011–2014 (in %)

Source: the author, on the basis of (NCBiR 2015, p. 71).

Figure 1 shows that in 2011 and 2014, entrepreneurs constituted the greatest group of beneficiaries of projects financed by the National Centre for Research and Development, while in 2012 and 2013 consortiums made up the largest group.

The resources allotted for financing science as one of the categories of public tasks have been distinguished in the Budget Act in section 28 – Science. Figure 2 presents the amount of budget resources spent in section 730 in 2014 by task.

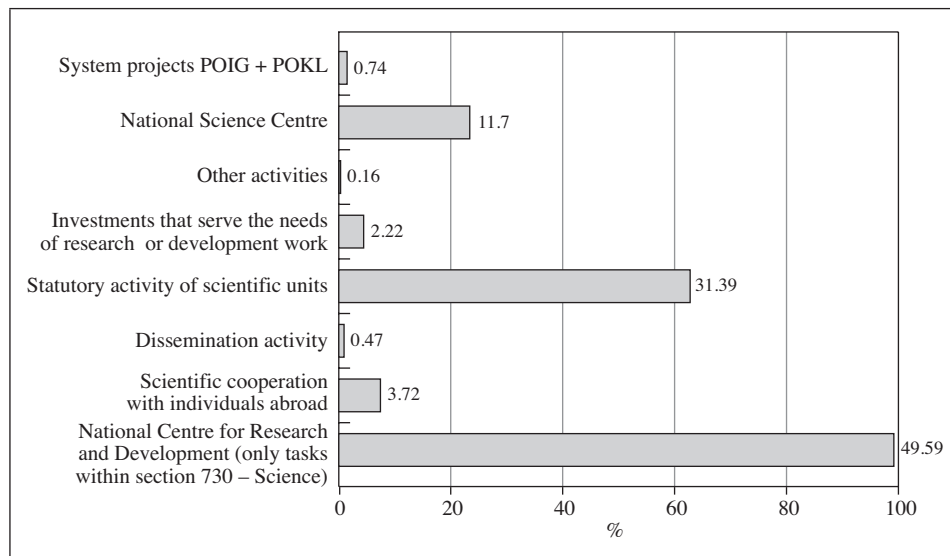


Fig. 2. Budget Expenses by Task in Section 730 – Science in 2014 (in %)

Source: the author, on the basis of (NCBiR 2015, p. 18).

In 2014, expenses of the National Centre for Research and Development constituted the greatest group of budget expenses in section 730 – approximately 50%. 32% of resources were spent for the statutory activity conducted by research units. The third largest category was the expenses of the National Science Centre.

### 3. On-going Evaluation vs. Partial Effectiveness

The source literature puts forward various approaches to interpreting the notion of effectiveness (Puczkowski 2014):

- an economic perspective – a relation between results and expenditures expressed through basic measures such as productivity, efficiency and profitability,
- a target perspective – evaluation of the degree to which the organisational objectives are attained, taking into account the economic perspective,
- a system perspective – evaluation of the use of organisational expenditures as well as the creation of specific relations with the environment,

– a comprehensive perspective – an ability of the organisation to reach its operational objectives.

The term “effectiveness” has various meanings, so the scope of analyses on effectiveness must be defined in the subject literature. The ambiguity of the notion of effectiveness was stressed by citing a general inability to ascribe explicit empirical content, Z. Kowalski emphasised the ambiguity of the notion of effectiveness. The method of defining effectiveness is always dependent on the context of analysis or additional comment, the effectiveness of substitution, effectiveness of investment. A similar view is expressed in studies by other authors: E. Pasour claimed that the notion of effectiveness was always subjective and should be measured and defined in relation to the objective of the analysis (Szymańska 2010).

J. Zieleniewski, in a paper on the effectiveness of scientific research, emphasises that it may constitute a subject of an analysis in the scope of intended and on-going scientific activity: “Which we can have influence on as well as which is determined as an *ex ante* view” (Zieleniewski 1966). One may perceive every action as consisting of partial actions, in relation to which Zieleniewski distinguishes the following forms of effectiveness:

- general integral – integral due to costs and results,
- relative integral – partial due to results but integral due to costs,
- general partial – general due to results, partial due to costs,
- relative partial – partial due to both results and costs,

For the present analysis, relative partial effectiveness was used.

Evaluation means “assessing value”. In the practice of Polish administration in the mid 1990s, evaluation meant simply “assessment”. However, it is really only one component of assessment. As E. Stern points out, it is also the field of knowledge that combines applied science and the practice of public policy (Olejniczak 2008). In the practice of international organisations and public sector institutions, evaluation is defined through its practical objective – improving the quality of public interventions (Szymańska 2010, p. 20). For successful evaluation it is fundamental to determine the object and subject of evaluation precisely, logically and cohesively, and to ask the key questions to be answered as well as the criteria which will be used during the evaluation (*PTE Evaluation Standards...* 2017). A fundamental function of evaluation is correcting possible disturbances and problems which may appear in the course of carrying out tasks.

On-going evaluation is characterised by the following features (<http://pte.org.pl/o-ewaluacji/>, accessed: 21.08.2017):

- it is carried out in the middle of implementation of an intervention,
- the products and results reached at this stage of analysis are analysed,
- it conducts the first assessment of a project’s and programme’s quality upon completion,

- it assesses the premises made at the programming stage (objectives, rates),
- it evaluates the context of a project's implementation,
- it may contribute to some modifications being made in the implementation of interventions and the updating the assumptions adopted.

These features show that there is a relationship between the on-going evaluation and assessment of a project's partial effectiveness. Performance of the on-going evaluation provides information on the output indicators.

## 4. Tools Used by the National Centre for Research and Development in Monitoring Partial Effectiveness

### 4.1. General Remarks

The National Centre for Research and Development conducts scientific research serving practical implementations. This field has been divided into six subfields (Table 1). To analyse on-going evaluation, the STRATEGMED programme was chosen. It is classified under the first area introduced in Table 1 – financing applied research. The main objective of the STRATEGMED programme is to eliminate diseases of affluence and regenerative medicine on the basis of scientific research and developmental works carried out in four fields: cardiology and heart surgery, oncology, neurology and the senses, and regenerative medicine (<http://www.ncbr.gov.pl/programy-strategiczne/profilaktyka-i-leczenie-chorob-cywilizacyjnych---strategmed/>, accessed: 24.08.2017).

Table 1. Financing Actions Taken in Scientific Research with Practical Use in 2015

Subfield	2015 Scheme (Thousands of PLN)	Performance in 2015 (Thousands of PLN)
Financing applied research	663,181	660,904
Financing R+D research and commercialisation of the results in subjects which can be applied in practice	1,632,619	1,178,358
Developing R+D staff competences and increasing awareness of the role science plays in economic development	26,948	24,139
R+D support in national defence	348,737	346,953
Financing the "Improvement of the working and safety conditions" multiannual programme – Stage 3	10,500	10,500
Support for common infrastructure of research units	1,850,252	1,748,471

Source: the author, on the basis of (NCBiR 2016, p. 102–126).

## 4.2. Partial Effectiveness in Relation to the NCRD Programmes

Cohesion and effectiveness are the most important criteria in evaluating the NCRD programmes. The first criterion – an internal examination – evaluates the programme for its transparency, clarity of concept and functionality of the solutions proposed. The external evaluation of the premises where the programme is conducted in relation to objectives of national scientific policy and innovative policy is determined in the National Research programme, among others. The second criterion – effectiveness – concerns the possibility of reaching the programme's objectives, assessing its qualifications and a proposed rating system ([http://archiwum.opolskie.pl/docs/9.\\_prof.\\_kurzydowski\\_x\\_n.pdf](http://archiwum.opolskie.pl/docs/9._prof._kurzydowski_x_n.pdf), accessed: 24.08.2017).

Within the first subfield defined in the field of scientific research serving for practical use – financing applied research – NCRD described the main objective as an increase of intensity of the research used for the purposes of the economic sector. The main rate of the first action has also been defined as the number of enterprises carrying out projects in the scope of applied research in cooperation with research units. The anticipated value of the rate at the end of 2015 was 380, while the actual performance came out at 502. This higher performance came thanks to two factors: redeployments in the timetable for the implementation of the project and a higher number of enterprises participating in new projects than had been expected according to assessments.

Financing applied research was accomplished in 2015 through (NCBiR 2016, p. 102–106):

- preparation of contest documentation,
- announcing a competition,
- call for applications,
- evaluation and selection of applications to be covered with financing,
- concluding agreements with development contractors,
- supervision of the execution and financing of projects,
- settlement of projects,
- *ex ante*, on-going, mid-term, *ex post*.

14,272 thousands of PLN was spent in 2015 within the resources allotted to service, supervision, dissemination of information on competitions and the popularisation of the effects of the measures taken within applied research as well as evaluation (NCBiR 2016, p. 102–106).

## 4.3. Partial Effectiveness in Relation to Beneficiaries of Resources

A fundamental project document that includes obligatory guidelines for on-going evaluation is the financing agreement between beneficiaries of

resources and the National Centre for Research and Development. It states that, to evaluate the project while it is being implemented, the beneficiary is obliged to cooperate with the Centre or an entity authorised by the Centre as well as to provide information concerning the project involved, particularly to submit information about the economic effects and other benefits incurred as a result of its implementation ([http://www.ncbr.gov.pl/gfx/ncbir/userfiles/\\_public/programy\\_strategiczne/strategmed/ii\\_konkurs/wzor\\_umowy\\_strategmed\\_iikonkurs.pdf](http://www.ncbr.gov.pl/gfx/ncbir/userfiles/_public/programy_strategiczne/strategmed/ii_konkurs/wzor_umowy_strategmed_iikonkurs.pdf), accessed: 23.08.2017).

Within the project implementation period, the contractor is obliged to submit to annual reports to NCRD, whose assessment is to determine whether ([http://www.ncbr.gov.pl/gfx/ncbir/userfiles/\\_public/programy\\_strategiczne/strategmed/ii\\_konkurs/wzor\\_umowy\\_strategmed\\_iikonkurs.pdf](http://www.ncbr.gov.pl/gfx/ncbir/userfiles/_public/programy_strategiczne/strategmed/ii_konkurs/wzor_umowy_strategmed_iikonkurs.pdf), accessed: 23.08.2017):

- 1) implementation of the project is in line with the agreement,
- 2) continuing the project will achieve the intended objectives.

In the event basic, industrial research or development works results indicate lack of possibilities for reaching project objectives, the contractor is obliged to immediately inform NCRD if the project objectives cannot be achieved, and then to apply to cease the project ([http://www.ncbr.gov.pl/gfx/ncbir/userfiles/\\_public/programy\\_strategiczne/strategmed/ii\\_konkurs/wzor\\_umowy\\_strategmed\\_iikonkurs.pdf](http://www.ncbr.gov.pl/gfx/ncbir/userfiles/_public/programy_strategiczne/strategmed/ii_konkurs/wzor_umowy_strategmed_iikonkurs.pdf), accessed: 23.08.2017).

Table 2. Selected Product Rates within the STRATEGMED Programme in 2015

Rates	Target Value	The Value Planned in 2015	The Value Reached in 2015
Number of publications on R+D works of programmes whose authors are members of research teams in periodicals included in the Science Citation Index	200	15	6
Number of new research teams formed to implement projects within the programme	40	9	56
Number of patent applications in PCT and EPO mode	50	1	3

Source: the author, developed on the basis of (NCBiR 2016, p. 102–106).

Table 2 presents demonstration product rates reported at the stage of implementation of the project in the annual report by beneficiary. Where there are deviations from the planned value, it is necessary to explain their causes.

The scientific project is implemented according to a schedule and activity-based budget – every research task has a budget and scope of work to perform. In partial settlement reports (annual), the beneficiary of a scientific project is obliged to indicate the level of work performance based on rates which are defined



at the planning stage in the application for a grant. The mechanism for evaluating an on-going project in material and cost scope allows for an analysis of the partial effectiveness of a scientific project.

## 5. Conclusion

The National Centre for Research and Development performs on-going evaluation at two levels. One directly concerns the beneficiaries in the scope of assessment of advancement of works as well as incurred costs. The other holistically assesses the effectiveness of all programmes implemented by the National Centre for Research and Development. At both levels, the Centre adopts indicators which are usually determined in periodic reports.

Effectiveness is one of the criteria of evaluation. In this aspect, effectiveness focuses on an analysis of the relationship between expenditures, costs, resources (human, financial, administrative) and the effects achieved. The on-going evaluation of a project enables the analysis of partial effectiveness. STRATEGMED, an NCRD programme, has clearly defined output indicators, and indicates the planned annual and target value.

With regard to the assessment of the effectiveness of partial effectiveness monitoring tools, it should be stated that the National Centre for Research and Development obliges beneficiaries of scientific projects to submit annual reports on project implementation. However, these reports should not only fulfill a reporting function but also a control function. The information contained in the annual report should be the starting point for further analysis in two main areas: compliance of project implementation with the financial agreement, and adequacy of the level of funds spent in relation to the level of the output indicators.

On-going evaluation makes it possible to assess the risk of a project moving away from its conceptual design. Therefore, monitoring the partial effectiveness of a scientific project protects against the risk of losing financing due to lack of implementation in operational objectives.

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## **Efektywność cząstkowa projektów naukowych finansowanych ze środków publicznych** (Streszczenie)

Jednym z rodzajów efektywności jest efektywność cząstkowa określana na etapie realizacji przedsięwzięcia z uwzględnieniem zaawansowania wykonania prac oraz poniesionych kosztów. Celem artykułu jest analiza oraz ocena skuteczności narzędzi monitorowania efektywności cząstkowej projektów naukowych, a w szczególności określenie zależności między ewaluacją bieżącą a efektywnością cząstkową w odniesieniu do projektów naukowych. Przedmiotem analizy są projekty naukowe finansowane w ramach programu Narodowego Centrum Badań i Rozwoju (NCBiR) – STRATEGMED. Analiza wydatkowania

środków przekazanych beneficjentom przez NCBiR została dokonana z zastosowaniem metody dynamicznej (strumieniowej), której istotą jest analiza przepływu środków pieniężnych między dwoma podmiotami. Metoda monograficzna posłużyła do analizy narzędzi monitorowania efektywności cząstkowej, umożliwiając szczegółowe zbadanie ich cech i elementów.

**Słowa kluczowe:** projekt, ewaluacja, efektywność, beneficjent.

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# Super-replication of European Options with Convex Payoff under Proportional Transaction Costs\*

## Abstract

The paper examines the super-replication of contingent claims in a discrete time financial market with proportional transaction costs. The sole assumption on stock price dynamics is that the returns are bounded. The class of path-dependent European options with nonnegative convex payoff functions is considered. It is proved that the pricing of this type of options can be studied through the pricing of a suitable binomial model. As a consequence, it is shown that the pricing algorithm, which is essentially a dynamic programming procedure on a tree, can be used when the set of possible scenarios is not finite.

**Keywords:** super-replication, transaction costs, model uncertainty, European options.

**JEL Classification:** G13, C61, D52.

## 1. Introduction

This article examines the super-replication of European options in a discrete time market model with transaction costs and volatility uncertainty. Such a problem naturally arises in various applications when European options are

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\* The research was financed from the funds granted to the Faculty of Finance and Law at Cracow University of Economics, within the framework of the subsidy for the maintenance of research potential.

priced. It is therefore not surprising that this issue has been studied fairly widely. The pricing and hedging of contingent claims under proportional transaction costs in the classical case has also been studied in a number of papers (Stettner 1997, Kociński 2004, Roux, Tokarz & Zastawniak 2008). Option pricing is closely related to the arbitrage under transaction costs studied in (Guasoni, Lépinette & Rásonyi 2012, Guasoni, Lépinette & Schachermayer 2008, Jouini & Kallal 1995, Schachermayer 2003).

A classical probabilistic model of a financial market consists of a measurable space  $(\Omega, \mathcal{F})$  and a probability measure  $\mathbb{P}$  determining the distribution of stock prices. In contrast, for this paper we studied a distribution not assumed to be (completely) known *a priori*. Our sole assumption on the stock price dynamic is that the absolute value of the log-returns is bounded from both below and above. The problem of super-replication in such a discrete-time model, with more general nonlinear transaction costs, is studied in (Bank, Dolinsky & Gökay 2016). The result, including the special case of a convex payoff profile, seems to be nontrivial when applied to the real market from point of view of the calculations. A. Roux, K. Tokarz and T. Zastawniak (2008) developed a pricing and hedging algorithm in the discrete setting under proportional transaction costs, which can be rapidly implemented on a computer, but does not allow for models with uncertain volatilities.

The aim of this paper is to show that pricing of convex European options can be reduced to studying the pricing of a suitable binomial model following the arguments of P. Bank, Y. Dolinsky and S. Gökay (2016) as well as A. Roux, K. Tokarz and T. Zastawniak (2008). The method presented leads to a feasible procedure which can be applied to options with convex payoffs in an uncertain volatility model with arbitrary proportional transaction costs.

## 2. Description of the Model

Let us consider a discrete time market model with a time horizon  $N \in \mathbb{N}$  with a riskless savings account  $B_n = 1, n = 0, \dots, N$  and a risky stock  $S_n > 0, n = 0, \dots, N$ . Let

$$X_n := \ln\left(\frac{S_n}{S_{n-1}}\right) \quad (1)$$

be the log-return for period  $n = 1, \dots, N$  such that

$$\underline{\sigma} \leq |X_n| \leq \bar{\sigma}, \quad n = 1, \dots, N \quad (2)$$

for some constants  $0 \leq \underline{\sigma} \leq \bar{\sigma} < \infty$ . Hence

$$S_n = s_0 \exp\left(\sum_{m=1}^n X_m\right), \quad n = 0, \dots, N. \quad (3)$$

Let

$$\Omega = \Omega_{\underline{\sigma}, \bar{\sigma}} := \left\{ \omega = (x_1, \dots, x_N) \in \mathbb{R}^N : \underline{\sigma} \leq |x_n| \leq \bar{\sigma}, \quad n = 1, \dots, N \right\} \quad (4)$$

be the path space with the canonical process

$$X_k(\omega) := x_k \quad \text{for } \omega = (x_1, \dots, x_N) \in \Omega \quad (5)$$

and the canonical filtration

$$\mathcal{F}_n := \sigma(X_1, \dots, X_n), \quad n = 0, \dots, N. \quad (6)$$

For every  $n = 0, \dots, N$  the cost of buying one share of the stock at time  $n$  is  $S_n(1 + \lambda)$ , where  $\lambda \in [0, \infty)$  and the amount received for selling one share at time  $n$  is  $S_n(1 - \mu)$  with  $\mu \in [0, 1)$ . A trading strategy is a pair  $(\eta, \theta)$  of predictable processes  $\eta_n, \theta_n$  representing positions in cash and stock respectively, at  $n = 0, \dots, N$ . The time  $n$  liquidation value  $\vartheta_n$  of a portfolio  $(\alpha, \beta)$  of cash and stock is defined as

$$\vartheta_n(\alpha, \beta) = \alpha + \beta^+(1 - \mu)S_n - \beta^-(1 + \lambda)S_n. \quad (7)$$

*Definition 1.* A pair  $(\eta, \theta)$  of predictable processes  $\eta_n, \theta_n$ , can be called a self-financing strategy, if

$$\vartheta_n(\eta_n - \eta_{n+1}, \theta_n - \theta_{n+1}) \geq 0 \quad (8)$$

for each  $n = 0, \dots, N$ , with  $\theta_0 = 0$ .

The class of self-financing strategies starting with initial capital shall be denoted by  $\eta_0$  by  $\Phi(\eta_0)$ .

*Definition 2.* The super-replication price of a European option  $F: \mathbb{R}_+^{N+1} \rightarrow \mathbb{R}_+$  is defined as

$$\pi_{\underline{\sigma}, \bar{\sigma}}(F) = \inf\{\eta_0 \in \mathbb{R} \mid \exists (\eta, \theta) \in \Phi(\eta_0): \vartheta_N(\eta_N(\omega) - F(S(\omega)), \theta_N(\omega)) \geq 0 \quad \forall \omega \in \Omega_{\underline{\sigma}, \bar{\sigma}}\}. \quad (9)$$

The aim here is to determine the minimal initial capital which allows one, by using self-financing strategy, to end up holding a solvent portfolio  $(\eta_N(\omega) - F(S(\omega)), \theta_N(\omega))$ , by delivering the payoff  $F(S)$  at time  $N$ . It should be emphasised that super-replication prices are not defined in an almost sure sense, as in the classical approach.

### 3. The Main Result

We consider the special case of a convex payoff profile and show that the super-replication price coincides with the one computed in the binomial model where the volatility always takes its maximal values. This result allows for the investigation to be continued in the finite path space and, consequently, for an algorithmic approach to the optimal super-replication of European options considered in (Roux, Tokarz & Zastawniak 2008) to be used.

*Theorem 3.* Suppose that the payoff function  $F: \mathbb{R}_+^{N+1} \rightarrow \mathbb{R}_+$  is convex. The super-replication price of any European option with payoff  $F$  is then given by

$$\pi_{\underline{\sigma}, \bar{\sigma}}(F) = \bar{\pi}(F)$$

where  $\bar{\pi}(F) = \pi_{\bar{\sigma}, \bar{\sigma}}(F)$  denotes the super-replication price of  $F(S)$  in the binomial model with volatility  $\bar{\sigma}$  and transaction costs  $\lambda, \mu$ .

A similar result for markets with friction is shown in (Bank, Dolinsky & Gökay 2016). Note that in (Bank, Dolinsky & Gökay 2016), the mark-to-market value rather than the liquidation value is considered. Moreover, this approach is based on a different definition of the self-financing strategy.

*Proof.* Note that  $\pi_{\underline{\sigma}, \bar{\sigma}}(F) \geq \bar{\pi}(F)$ . It suffices to show that for any  $\varepsilon > 0$  there exists a self-financial strategy  $(\eta, \theta)$  which super-replicates  $F(S)$  in every scenario  $\omega \in \Omega$  with initial position  $(\varepsilon + \bar{\pi}(F), 0)$ . Consider the binomial model with volatility  $\bar{\sigma}$ . Let  $\bar{\Omega} = \{-1, 1\}^N$  be the path space with canonical process  $\bar{X}_n(\bar{\omega}) := \bar{x}_n$  for  $\bar{\omega} = (\bar{x}_1, \dots, \bar{x}_N) \in \bar{\Omega}$  and the stock price evolution  $\bar{S}_0 = s_0$  and  $\bar{S}_n = \bar{S}_{n-1} \exp(\bar{\sigma} \bar{X}_n)$ ,  $n = 1, \dots, N$ . Clearly, the canonical filtration

$$\bar{\mathcal{F}}_n := \sigma(\bar{X}_1, \dots, \bar{X}_n), \quad n = 0, \dots, N \quad (10)$$

coincides with the one generated by  $\bar{S} = (\bar{S}_n)_{n=0, \dots, N}$ . By the definition of  $\bar{\pi}(F)$  there is  $(\bar{\mathcal{F}}_n)_{n=0, \dots, N}$ -predictable process  $(\bar{\eta}, \bar{\theta})$  such that with  $\bar{\theta}_0 = 0$ . We therefore have

$$\vartheta_0(\varepsilon + \bar{\pi}(F) - \bar{\eta}_1, \bar{\theta}_0 - \bar{\theta}_1) \geq 0 \quad (11)$$

$$\vartheta_n(\bar{\eta}_n - \bar{\eta}_{n+1}, \bar{\theta}_n - \bar{\theta}_{n+1}) \geq 0, \quad n = 1, \dots, N-1 \quad (12)$$

$$\vartheta_N(\bar{\eta}_N - F(\bar{S}), \bar{\theta}_N) \geq 0 \quad (13)$$

everywhere on  $\bar{\Omega}$ . In view of inequalities (2) for any  $\omega \in \Omega$  and  $n = 1, \dots, N$  there are unique weights  $w_n^{(+1)}(\omega), w_n^{(-1)}(\omega) \geq 0$  with  $w_n^{(+1)}(\omega) + w_n^{(-1)}(\omega) = 1$  such that

$$e^{X_n(\omega)} = w_n^{(+1)}(\omega)e^{\bar{\sigma}} + w_n^{(-1)}(\omega)e^{-\bar{\sigma}}. \quad (14)$$

Observe that for weights

$$w_n^{\bar{\omega}^n}(\omega) := \prod_{m=1}^n w_n^{(\bar{\omega}_m^n)}(\omega), \quad \bar{\omega}^n = (\bar{\omega}_1^n, \dots, \bar{\omega}_n^n) \in \{-1, 1\}^n \quad (15)$$

we have

$$\sum_{\bar{\omega}^n \in \{-1, 1\}^n} w_n^{\bar{\omega}^n}(\omega) = \prod_{m=1}^n (w_m^{(+1)}(\omega) + w_m^{(-1)}(\omega)) = 1 \quad (16)$$

for  $n = 1, \dots, N$  and

$$w_n^{\bar{\omega}^n}(\omega) = w_n^{\bar{\omega}^n}(\omega) \prod_{m=n+1}^N (w_m^{(+1)}(\omega) + w_m^{(-1)}(\omega)) = \sum_{\bar{\omega}^{N-n} \in \{-1, 1\}^{N-n}} w_N^{(\bar{\omega}^n, \bar{\omega}^{N-n})}(\omega). \quad (17)$$

By using (3), (14), (17) and the adaptedness of  $\bar{S}$  we get the following representation:

$$\begin{aligned} S_n(\omega) &= s_0 \prod_{m=1}^n (w_m^{(+1)}(\omega)e^{\bar{\sigma}} + w_m^{(-1)}(\omega)e^{-\bar{\sigma}}) \\ &= \sum_{\bar{\omega}^n \in \{-1, 1\}^n} \bar{S}_n(\bar{\omega}^n, 1, \dots, 1) w_n^{\bar{\omega}^n}(\omega) = \sum_{\bar{\omega} \in \bar{\Omega}} \bar{S}_n(\bar{\omega}) w_N^{\bar{\omega}}(\omega) \end{aligned} \quad (18)$$

for any  $n = 1, \dots, N$ ,  $\omega \in \Omega$ . Now consider the pair  $(\eta, \theta)$  of predictable processes  $\eta_n, \theta_n$ :

$$\eta_n(\omega) := \begin{cases} \varepsilon + \bar{\pi}(F), & n = 0 \\ \bar{\eta}_1, & n = 1 \\ \sum_{\bar{\omega}^{n-1} \in \{-1, 1\}^{n-1}} \bar{\eta}_n(\bar{\omega}^{n-1}, 1, \dots, 1) w_{n-1}^{\bar{\omega}^{n-1}}(\omega), & n = 2, \dots, N \end{cases} \quad (19)$$

$$\theta_n(\omega) := \begin{cases} 0, & n = 0 \\ \bar{\theta}_1, & n = 1 \\ \frac{1}{S_{n-1}(\omega)} \sum_{\bar{\omega}^{n-1} \in \{-1, 1\}^{n-1}} \bar{\theta}_n \bar{S}_{n-1}(\bar{\omega}^{n-1}, 1, \dots, 1) w_{n-1}^{\bar{\omega}^{n-1}}(\omega), & n = 2, \dots, N. \end{cases} \quad (20)$$

First we prove that the strategy  $(\eta, \theta)$  is self-financing. In view of (11), (7), (16), (19) and (20) we have

$$\begin{aligned} 0 &\leq \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \vartheta_0(\varepsilon + \bar{\pi}(F) - \bar{\eta}_1(\bar{\omega}), \bar{\theta}_0(\bar{\omega}) - \bar{\theta}_1(\bar{\omega})) \\ &= \vartheta_0(\varepsilon + \bar{\pi}(F) - \bar{\eta}_1(\bar{\omega}), \bar{\theta}_0(\bar{\omega}) - \bar{\theta}_1(\bar{\omega})) \\ &= \vartheta_0(\varepsilon + \bar{\pi}(F) - \eta_1(\omega), \theta_0(\omega) - \theta_1(\omega)). \end{aligned} \quad (21)$$



Observe that

$$\sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \bar{\eta}_n(\bar{\omega}) = \sum_{\bar{\omega}^{n-1} \in \{-1, 1\}^{n-1}} w_{n-1}^{\bar{\omega}^{n-1}}(\omega) \bar{\eta}_n(\bar{\omega}^{n-1}, 1, \dots, 1) = \eta_n(\omega) \quad (22)$$

for any  $n = 1, \dots, N$ ,  $\omega \in \Omega$ . Since  $\bar{\theta}$  is predictable with respect to the filtration  $(\bar{\mathcal{F}}_n)_{n=0, \dots, N}$  and  $\bar{S}$  is  $(\bar{\mathcal{F}}_n)_{n=0, \dots, N}$ -adapted we have

$$\begin{aligned} \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \bar{\theta}_{n+1}(\bar{\omega}) \bar{S}_n(\bar{\omega}) &= \sum_{\bar{\omega}^n \in \{-1, 1\}^n} w_n^{\bar{\omega}^n}(\omega) (\bar{\theta}_{n+1} \bar{S}_n)(\bar{\omega}^n, 1, \dots, 1) = \\ &= \theta_{n+1}(\omega) S_n(\omega) \end{aligned} \quad (23)$$

for any  $n = 1, \dots, N$ ,  $\omega \in \Omega$  and

$$\begin{aligned} \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \bar{\theta}_n(\bar{\omega}) \bar{S}_n(\bar{\omega}) &= \sum_{\bar{\omega}^n \in \{-1, 1\}^n} w_n^{\bar{\omega}^n}(\omega) (\bar{\theta}_n \bar{S}_n)(\bar{\omega}^n, 1, \dots, 1) = \\ &= \sum_{\bar{\omega}^{n-1} \in \{-1, 1\}^{n-1}} \sum_{\bar{x} \in \{-1, 1\}} w_{n-1}^{\bar{\omega}^{n-1}}(\omega) w_n^{\bar{x}}(\omega) (\bar{\theta}_n \bar{S}_n)(\bar{\omega}^{n-1}, \bar{x}, \dots, 1) = \\ &= \sum_{\bar{\omega}^{n-1} \in \{-1, 1\}^{n-1}} w_{n-1}^{\bar{\omega}^{n-1}}(\omega) (\bar{\theta}_n \bar{S}_{n-1})(\bar{\omega}^{n-1}, 1, \dots, 1) \left[ w_n^{(+)}(\omega) e^{\bar{\sigma}} + w_n^{(-)}(\omega) e^{-\bar{\sigma}} \right] = \\ &= \theta_n(\omega) S_{n-1}(\omega) e^{X_n(\omega)} = \theta_n(\omega) S_n(\omega) \end{aligned} \quad (24)$$

for any  $n = 2, \dots, N-1$ ,  $\omega \in \Omega$ . Now, using (12) and (7) we obtain

$$\begin{aligned} 0 &\leq \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \vartheta_n(\bar{\eta}_n(\bar{\omega}) - \bar{\eta}_{n+1}(\bar{\omega}), \bar{\theta}_n(\bar{\omega}) - \bar{\theta}_{n+1}(\bar{\omega})) = \\ &= \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) (\bar{\eta}_n(\bar{\omega}) - \bar{\eta}_{n+1}(\bar{\omega})) + \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) (\bar{\theta}_n(\bar{\omega}) - \bar{\theta}_{n+1}(\bar{\omega})) \bar{S}_n(\bar{\omega}) + \\ &\quad - \mu \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) (\bar{\theta}_n(\bar{\omega}) - \bar{\theta}_{n+1}(\bar{\omega}))^+ \bar{S}_n(\bar{\omega}) + \\ &\quad - \lambda \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) (\bar{\theta}_n(\bar{\omega}) - \bar{\theta}_{n+1}(\bar{\omega}))^- \bar{S}_n(\bar{\omega}) \end{aligned} \quad (25)$$

for any  $n = 1, \dots, N-1$ ,  $\omega \in \Omega$ , which in conjunction with (22), (23), (24) and the convexity of the positive and the negative part entails the inequality

$$0 \leq \vartheta_n(\eta_n(\omega) - \eta_{n+1}(\omega), \theta_n(\omega) - \theta_{n+1}(\omega)) \quad (26)$$

for any  $n = 1, \dots, N-1$ ,  $\omega \in \Omega$ . The super-replication condition

$$\vartheta_N(\eta_N(\omega) - F(S), \theta_N(\omega)) \geq 0 \quad (27)$$

for  $\omega \in \Omega$  remains to be proved. By definition of  $\vartheta_n$  and (13) we have

$$\begin{aligned}
0 &\leq \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \vartheta_N(\bar{\eta}_N(\bar{\omega}) - F(\bar{S}(\bar{\omega})), \bar{\theta}_N(\bar{\omega})) = \\
&= \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) (\bar{\eta}_N(\bar{\omega}) - F(\bar{S}(\bar{\omega}))) + \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \bar{\theta}_N(\bar{\omega}) \bar{S}_N(\bar{\omega}) + \\
&- \mu \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) (\bar{\theta}_N(\bar{\omega}))^+ \bar{S}_N(\bar{\omega}) - \lambda \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) (\bar{\theta}_N(\bar{\omega}))^- \bar{S}_N(\bar{\omega}).
\end{aligned} \tag{28}$$

Note that

$$\begin{aligned}
\sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) F(\bar{S}(\bar{\omega})) &\geq F\left(\sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \bar{S}_0(\bar{\omega}), \dots, \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \bar{S}_N(\bar{\omega})\right) = \\
&= F(S_0(\omega), \dots, S_N(\omega)) = F(S(\omega))
\end{aligned} \tag{29}$$

due to the convexity of  $F: \mathbb{R}_+^{N+1} \rightarrow \mathbb{R}_+$  and (18). Similarly,

$$\mu \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) (\bar{\theta}_N(\bar{\omega}))^+ \bar{S}_N(\bar{\omega}) \geq \mu \theta_N^+(\omega) S_N(\omega) \tag{30}$$

$$\lambda \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) (\bar{\theta}_N(\bar{\omega}))^- \bar{S}_N(\bar{\omega}) \geq \lambda \theta_N^-(\omega) S_N(\omega). \tag{31}$$

As a consequence,

$$\begin{aligned}
0 &\leq \sum_{\bar{\omega} \in \bar{\Omega}} w_N^{\bar{\omega}}(\omega) \vartheta_N(\bar{\eta}_N(\bar{\omega}) - F(\bar{S}(\bar{\omega})), \bar{\theta}_N(\bar{\omega})) \\
&\leq \vartheta_N(\eta_N(\omega) - F(S(\omega)), \theta_N(\omega)).
\end{aligned} \tag{32}$$

This shows that the self-financing strategy  $(\eta, \theta)$  super-replicates  $F(S)$  with  $\eta_0 = \varepsilon + \bar{\pi}(F)$ , which completes the proof.

## 4. Consequences and Further Generalisations

In setup used for this paper, all possible stock price evolutions which respect the specified volatility bounds are uncountable, but in view of Theorem 3, from Theorem 4.2 in (Roux, Tokarz & Zastawniak 2008), corollary 4 is obtained.

*Corollary 4.* The super-replication price of a European option with convex payoff  $F(S)$  is given by

$$\pi_{\underline{\sigma}, \bar{\sigma}}(F) = \max_{(\mathbb{P}, \hat{S}) \in \mathcal{P}} \mathbb{E}_{\mathbb{P}}(F(S)) = \max_{x \in \mathbb{R}} Z_0^x - (1 + \lambda) S_0,$$

where  $\mathcal{P}$  denotes the set of pairs  $(\mathbb{P}, \hat{S})$  such that  $\mathbb{P}$  is probability measure on  $\bar{\Omega}$  and  $\hat{S}$  is a martingale under  $\mathbb{P}$  satisfying

$$S_n(1 - \mu) \leq \widehat{S}_n \leq S_n(1 + \lambda)$$

for any  $n = 0, \dots, N$ ,  $Z_0$  is the polyhedral proper convex function constructed as follows:

– we put

$$Z_N^x = \widetilde{Z}_N^x = \begin{cases} F(S) + x & \text{if } x \in [S_N(1 - \mu), S_N(1 + \lambda)] \\ -\infty & \text{if } x \notin [S_N(1 - \mu), S_N(1 + \lambda)], \end{cases}$$

– for any  $n = 0, \dots, N - 1$  we take

$$\widetilde{Z}_n(\overline{\omega}^n) = \text{cap}\{Z_{n+1}(\overline{\omega}^{n+1}): \overline{\omega}^{n+1} \in \{(\overline{\omega}^n, 1), (\overline{\omega}^n, -1)\}\}$$

(the concave  $\text{cap}\{f_1, \dots, f_n\}$  of functions  $f_1, \dots, f_n: \mathbb{R} \rightarrow \mathbb{R} \cup \{-\infty\}$  is defined as the smallest concave function  $h$  such that  $h \geq \max\{f_1, \dots, f_n\}$ )

and

$$Z_n^x = \begin{cases} \widetilde{Z}_n^x & \text{if } x \in [S_n(1 - \mu), S_n(1 + \lambda)] \\ -\infty & \text{if } x \notin [S_n(1 - \mu), S_n(1 + \lambda)]. \end{cases}$$

This method makes it possible to price the European option algorithmically, as was studied in (Roux, Tokarz & Zastawniak 2008).

This paper has examined proportional transaction costs, the results of which can be extended to convex transaction costs. Further extension will consider multi-asset cases and generalize the results in this direction.

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### **Super-replikacja opcji europejskich o wypukłej funkcji wypłaty przy proporcjonalnych kosztach transakcji**

(Streszczenie)

W artykule przedstawiono super-replikację instrumentów pochodnych na rynku finansowym z czasem dyskretnym z proporcjonalnymi kosztami transakcji. O dynamice cen akcji zakładano wyłącznie, że stopy zwrotu są ograniczone. Rozpatrywano klasę europejskich opcji zależnych od trajektorii, o nieujemnych, wypukłych funkcjach wypłaty. Udowodniono, że problem wyceny tego typu opcji można zredukować do wyceny w odpowiednim modelu dwumianowym. Pokazano zatem, że algorytm wyceny, będący w istocie algorytmem programowania dynamicznego na drzewie, może zostać wykorzystany, w przypadku gdy przestrzeń możliwych scenariuszy nie jest skończona.

**Słowa kluczowe:** super-replikacja, koszty transakcji, niepewność modelu, opcje europejskie.

ISSN 1898-6447

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